Effects of skill multimap misspecification on parameter estimates in probabilistic knowledge structures

Pasquale Anselmi, Luca Stefanutti, Egidio Robusto

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Subject of the presentation

- The `pks` package was used to investigate the effects of skill multimap misspecification on
  - the estimation of the problem parameters (careless errors and lucky guesses)
  - the recovery of the correct competence state
- Both maximum-likelihood estimates and minimum-discrepancy estimates of the parameters are considered

Knowledge space theory: Main concepts

- Knowledge domain identified with a set Q of problems, each of which has a correct response
- Knowledge state as the collection $K \subseteq Q$ of all problems in Q that a student is capable of solving
- Knowledge structure as a collection $\mathcal{K}$ of knowledge states K containing at least the the empty set $\emptyset$ and the full set Q

Skills underlying the problems: The competency model

- The assumption is made that there is a set S of discrete skills, which are relevant to solve the problems
- A skill multimap associates each problem $q \in Q$ with a nonempty collection $\mu(q)$ of nonempty subsets of skills
- Each subset $C \in \mu(q)$ represents a *competency* for q
- The knowledge state delineated by the subset of skills $X \subseteq S$ is specified by $K = \{q \in Q : C \subseteq X \text{ for some } C \in \mu(q)\}$

The competency model: Example

- Let $Q = \{1, 2, 3, 4\}, S = \{a, b, c\}$, and let μ be defined by

$$\mu(1) = \{\{a, b\}, \{a, c\}\},\ \mu(2) = \{\{c\}\},$$
$$\mu(3) = \{\{a\}, \{b, c\}\},\ \mu(4) = \{\{b\}\}$$

- The knowledge states delineated by each subset of skills are

X	K	X	K
$\{\}$	$\emptyset$	$\{a, b\}$	$\{1, 3, 4\}$
$\{a\}$	$\{3\}$	$\{a, c\}$	$\{1, 2, 3\}$
$\{b\}$	$\{4\}$	$\{b, c\}$	$\{2, 3, 4\}$
$\{c\}$	$\{2\}$	$\{a, b, c\}$	Q

- The knowledge structure is

$$\mathcal{K} = \{\emptyset, \{2\}, \{3\}, \{4\}, \{1, 2, 3\}, \{1, 3, 4\}, \{2, 3, 4\}, Q\}$$

Probabilistic knowledge structures

- The knowledge structure is a deterministic model of the organization of knowledge in a certain domain.

- There may be not perfect correspondence between the knowledge state of a student and his response pattern (careless errors and lucky guesses)

- The knowledge states may occur with different frequencies within the population of reference

- The probability of the response pattern R is specified by

$$P(R) = \sum_{K \in \mathcal{K}} P(R|K)P(K),$$

Probabilistic knowledge structures

- It is assumed that
  - the responses to the problems are locally independent, given the knowledge state of the student
  - the response to each problem q only depends on the careless error β_q and the lucky guess η_q probabilities

- Therefore, the probability of the response pattern R given the knowledge state K takes on the form

$$P(R|K) = \left(\prod_{q \in K \setminus R} \beta_q\right)\left(\prod_{q \in K \cap R} (1 - \beta_q)\right)\left(\prod_{q \in R \setminus K} \eta_q\right)\left(\prod_{q \in \overline{R \cup K}} (1 - \eta_q)\right),$$

where $\overline{R \cup K} = Q \setminus (R \cup K)$

Effects of misspecification of the problem-skill association: What is known

- A systematic investigation conducted on the DINA model showed that (Rupp & Templin, 2008):
  - the deletion of a skill from a problem causes the overestimation of its careless error probability
  - the addition of a skill to a problem causes an overestimation of its lucky guess probability

- The misspecifications:
  - have predominantly local effects
  - may affect the assessement of knowledge

Effects of misspecification of the problem-skill association: What has still to be investigated

- What are the effects on competency models? An investigation conducted on the multiple-strategy DINA model suggests that (de la Torre & Douglas, 2008):
  - the omission of a competency from a problem causes an overestimation of its lucky guess probability
  - the inclusion of a competency to a problem causes an overestimation of its careless error probability
- What are the effects on minimum-discrepancy (Wickelmaier & Heller, 2003) estimates of the parameters

The study

- Aim: Investigating the effects of different misspecifications of the skill multimap on
  - the careless error and lucky guess estimates of the problems
  - the recovery of the correct competence states
- Simulation of the data:
  - A collection with 20 problems was considered, and 5 skills were set to underlie them
  - Via the competency model, the problems were associated with the competencies according to the following skill multimap

The study

Problem	Competencies	Problem	Competencies
1	$\{a\}$	11	$\{a,d\}$
2	$\{b\}$	12	$\{b,c,e\}$
3	$\{c\}$	13	$\{a,b,c\}$
4	$\{d\}$	14	$\{a,d,e\}$,$\{b,d,e\}$
5	$\{e\}$	15	$\{a,c,d\}$
6	$\{a,b\}$,$\{a,c\}$	16	$\{b,c,d\}$
7	$\{a,e\}$	17	$\{a,b,c,d\}$,$\{a,c,d,e\}$
8	$\{c,e\}$	18	$\{a,b,d,e\}$
9	$\{d,e\}$,$\{b,e\}$	19	$\{b,c,d,e\}$
10	$\{b,c\}$	20	$\{a,b,c,d,e\}$

Note. Letters from a to e refer to the five skills.

- The knowledge structure delineated by the skill multimap was used to simulate a data set containing 10,000 response patterns
- The β and η parameters were randomly generated between 0 and .05
- The $P(K)$ probabilities were randomly generated

The study

- Types of misspecifications:
  - Deletion of a skill
  - Addition of a skill
  - Deletion of a skill and addition of another
  - Deletion of a competency
  - Addition of a competency
- Aspects of the misspecifications which are considered:
  - They regard problems associated with one or two competencies
  - They create or not problems which are clones of each other
- The models which derive from the misspecifications were estimated on the simulated data set

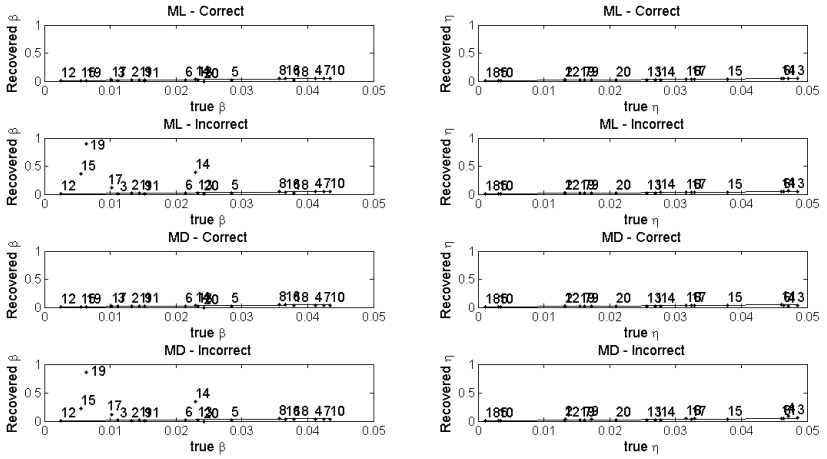
Results: Deletion of a skill which does not create clones

Problem	N competencies	True	Modified
14	2	$\{b, d, e\}$	$\{b, d\}$
15	1	$\{a, c, d\}$	$\{c, d\}$
17	2	$\{a, c, d, e\}$	$\{a, c, e\}$
19	1	$\{b, c, d, e\}$	$\{c, d, e\}$

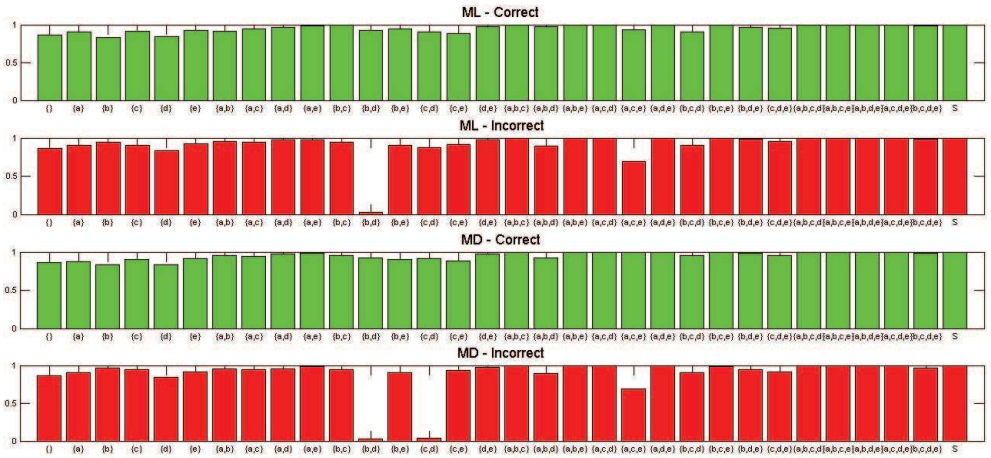
Note. Letters from a to e refer to the five skills.

Results: Deletion of a skill which does not create clones

- ML: $\beta_{14} = .39; \beta_{15} = .37; \beta_{17} = .11; \beta_{19} = .89$
- MD: $\beta_{14} = .34; \beta_{15} = .22; \beta_{17} = .11; \beta_{19} = .86$



Results: Deletion of a skill which does not create clones



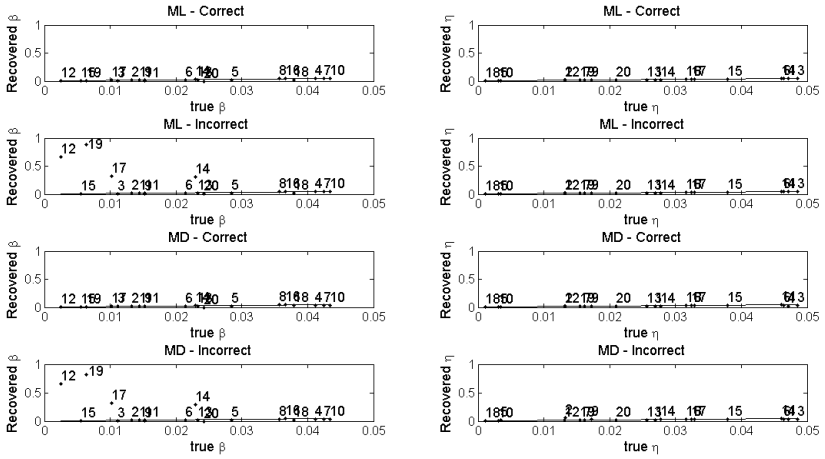
Results: Deletion of a skill which creates clones

Problem	N competencies	True	Modified	Clone problem
12	1	$\{b, c, e\}$	$\{c, e\}$	8
14	2	$\{a, d, e\}$	$\{d, e\}$	9
17	2	$\{a, c, d, e\}$	$\{a, c, d\}$	15
19	1	$\{b, c, d, e\}$	$\{b, d, e\}$	14

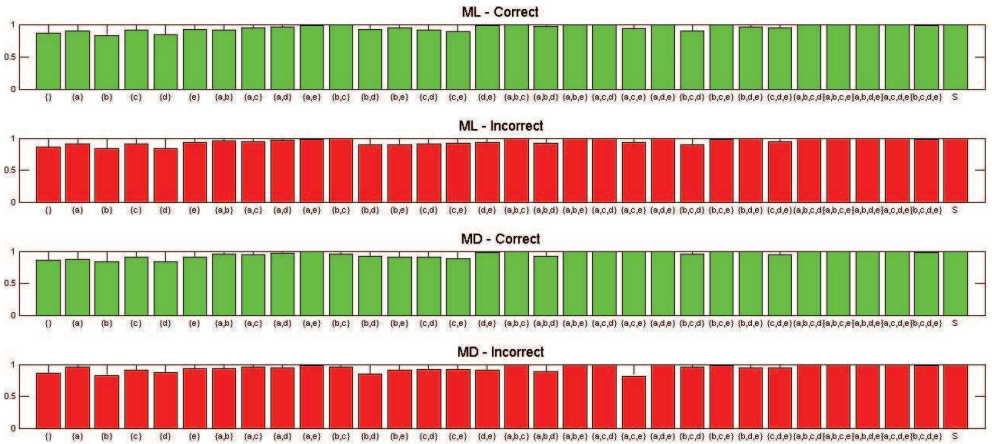
Note. Letters from a to e refer to the five skills.

Results: Deletion of a skill which creates clones

- ML: $\beta_{12} = .67; \beta_{14} = .31; \beta_{17} = .05; \beta_{19} = .88$
- MD: $\beta_{12} = .66; \beta_{14} = .29; \beta_{17} = .05; \beta_{19} = .82$



Results: Deletion of a skill which creates clones



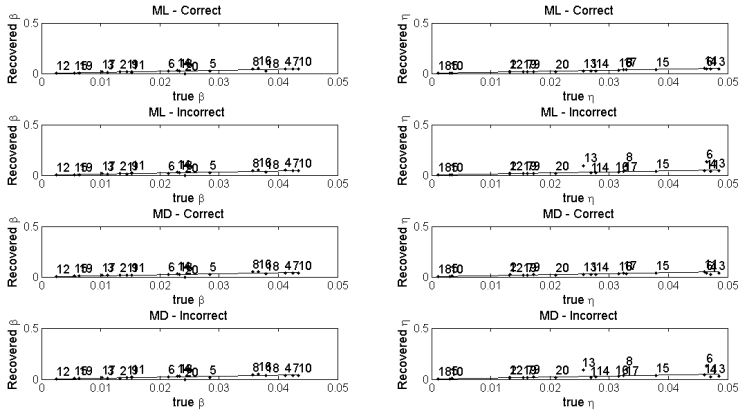
Results: Addition of a skill which does not create clones

Problem	N competencies	True	Modified
6	2	$\{a, b\}$	$\{a, b, d\}$
8	1	$\{c, e\}$	$\{a, c, e\}$
13	1	$\{a, b, c\}$	$\{a, b, c, e\}$

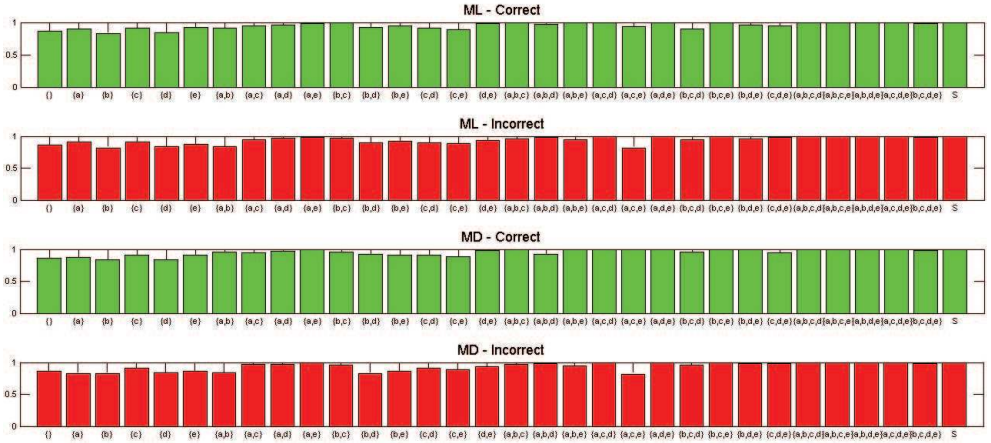
Note. Letters from a to e refer to the five skills.

Results: Addition of a skill which does not create clones

- ML: $\eta_6 = .14; \eta_8 = .10; \eta_{13} = .09$
- MD: $\eta_6 = .13; \eta_8 = .10; \eta_{13} = .09$



Results: Addition of a skill which does not create clones



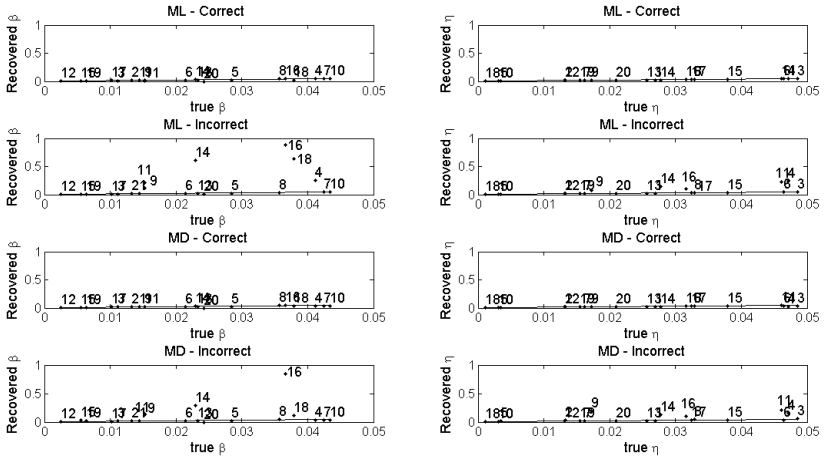
Results: Deletion and addition of skills which do not create clones

Problem	N competencies	True	Modified
9	2	$\{b, e\}$	$\{b, d\}$
11	1	$\{a, d\}$	$\{c, d\}$
14	2	$\{a, d, e\}$	$\{a, b, d\}$
16	1	$\{b, c, d\}$	$\{c, d, e\}$

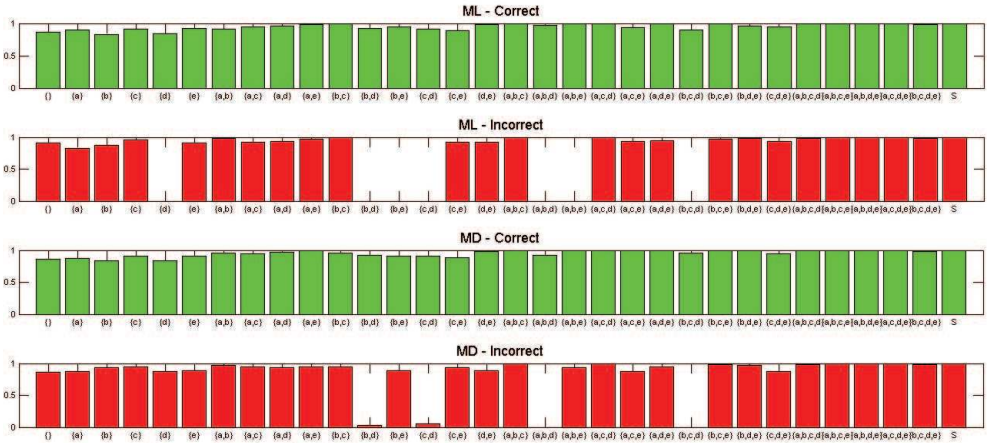
Note. Letters from a to e refer to the five skills.

Results: Deletion and addition of skills which do not create clones

- ML: $\beta_9 = .12; \beta_{11} = .22; \beta_{14} = .60; \beta_{16} = .88 - \eta_9 = .08; \eta_{11} = .22; \eta_{14} = .14; \eta_{16} = .10$
- MD: $\beta_9 = .14; \beta_{11} = .14; \beta_{14} = .30; \beta_{16} = .84 - \eta_9 = .19; \eta_{11} = .21; \eta_{14} = .13; \eta_{16} = .10$



Results: Deletion and addition of skills which do not create clones



- Overestimation of the problem parameters but not relevant biases on the recovery of the competence states when the modification creates clones.

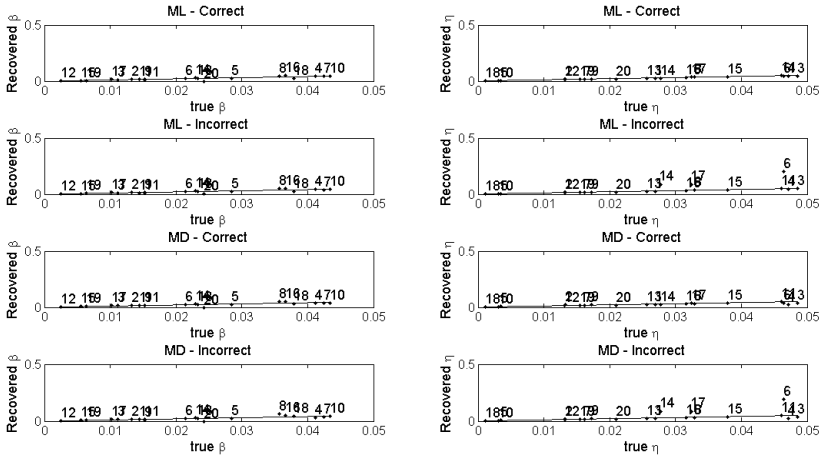
Results: Deletion of a competency

Problem	N competencies	True	Modified
6	2	$\{a, b\}$,$\{a, c\}$	$\{a, b\}$
14	2	$\{a, d, e\}$,$\{b, d, e\}$	$\{a, d, e\}$
17	2	$\{a, b, c, d\}$,$\{a, c, d, e\}$	$\{a, b, c, d\}$

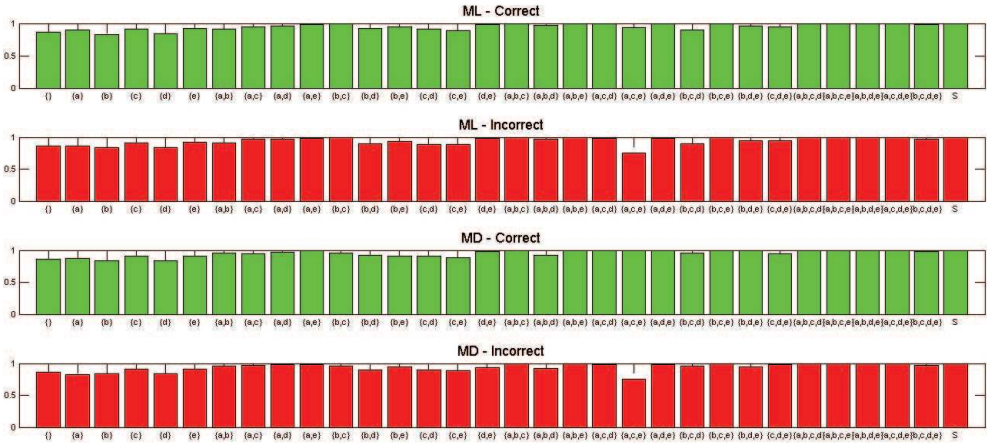
Note. Letters from a to e refer to the five skills.

Results: Deletion of a competency

- ML: $\eta_6 = .20; \eta_{14} = .09; \eta_{17} = .09$
- MD: $\eta_6 = .19; \eta_{14} = .09; \eta_{17} = .09$



Results: Deletion of a competency



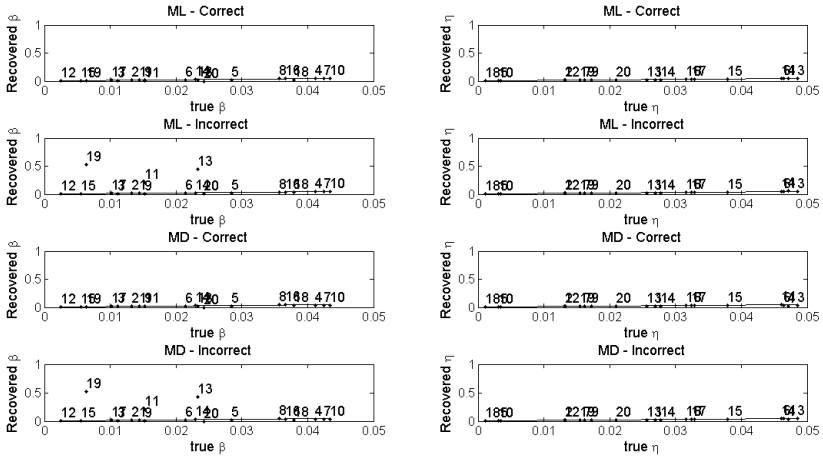
Results: Addition of a competency which does not create clones

Problem	N competencies	True	Modified
11	1	$\{a, d\}$	$\{a, d\}, \{c, d\}$
13	1	$\{a, b, c\}$	$\{a, b, c\}, \{a, b, e\}$
19	1	$\{b, c, d, e\}$	$\{b, c, d, e\}, \{a, b, c, e\}$

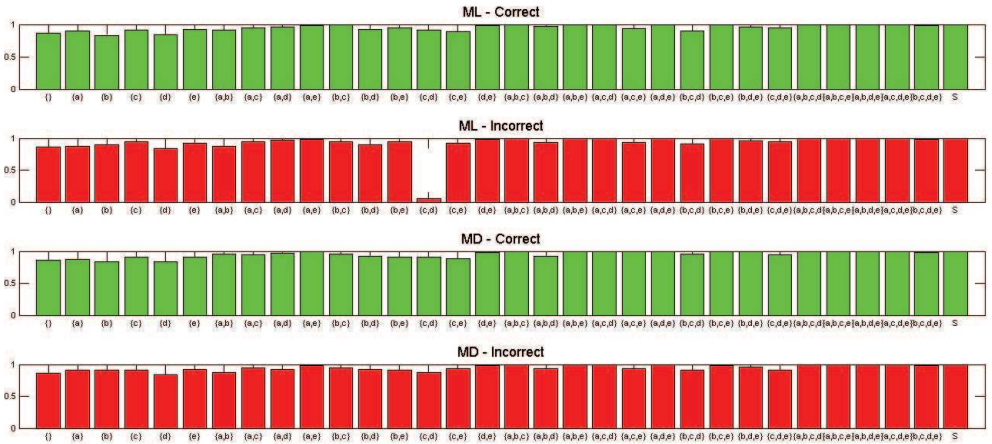
Note. Letters from a to e refer to the five skills.

Results: Addition of a competency which does not create clones

- ML: $\beta_{11} = .21; \beta_{13} = .44; \beta_{19} = .53$
- MD: $\beta_{11} = .20; \beta_{13} = .42; \beta_{19} = .52$



Results: Addition of a competency which does not create clones



Conclusions

- The misspecifications affect both the maximum-likelihood and the minimum-discrepancy estimates of the parameters. Hovewer, in the latter
  - the overestimations of careless error and lucky guess parameters are in general lower
  - the recovery of the correct competence state is in general better
- The effects of misspecifications on the careless error parameters are greater in the problems associated with a single competency
- In presence of problems which are clones of each other
  - the effects of misspecifications continue to be predominantly local
  - the probability of recovering the correct competence state increases if one of the clone problems is correctly associated with the skills

Future investigations

- Analyzing the effects of misspecifications when taking into account
  - different skill multimaps
  - different levels of noise in the data set
  - problems that are clones in the correct skill multimap

Thanks for your attention!

pasquale.anselmi@unipd.it