EBERHARD KARLS UNIVERSITÄT TÜBINGEN

Dimensionality of the Perceptual Space of Achromatic Colors

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Research Methods and Mathematical Psychology

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Outline

Color perception

- We have a tendency to treat color as a property of objects
- Experienced color is neither a property of objects, nor a property of light
- The physical or physiological quantifications of color do not fully explain the psychophysical perception of color appearance

Color perception

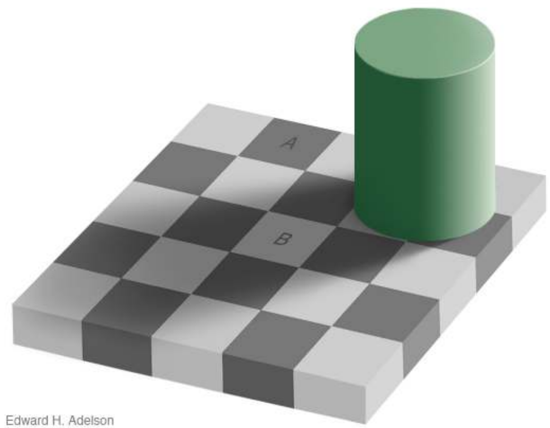
- We have a tendency to treat color as a property of objects
- Experienced color is neither a property of objects, nor a property of light
- The physical or physiological quantifications of color do not fully explain the psychophysical perception of color appearance

- In this talk we will only focus on *achromatic colors*

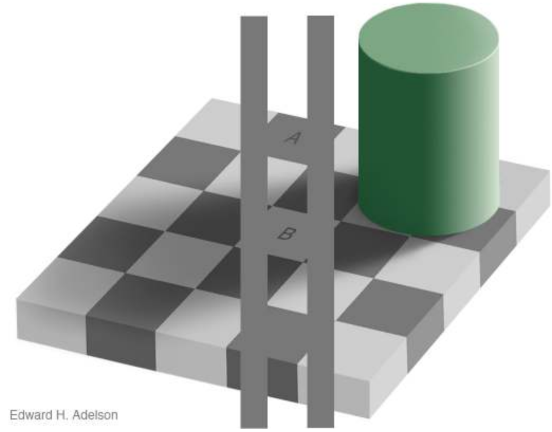
Dimensionality of the perceptual space of achromatic colors

- Traditional view assumes that achromatic color perception may be represented by a unidimensional achromatic color space (ranging from white to black)
- Logvinenko & Maloney (2006) and Niederée (2010) present recent evidence that this representation is at least two-dimensional
- Up to now there is no systematic investigation of the structure of the perceptual space of achromatic colors
- Our experiments aim at a characterization of the perceptual space of achromatic colors for individual observers

Demonstration



Demonstration



Ratio principle

ratio principle: Lightness is determined merely by the luminance ratio between a given surface and its surround, without reference to the level of illumination. (Gilchrist, 2006, p. 82)

- Prominent explanation of experimental results where subjects had to match two centers presented in different surrounds (postulated by Wallach, 1948)
- Ratio principle postulates that centers will be adjusted until ratio between center and surround is (nearly) identical for both configurations
- Infields will then be perceived as metameric (being of the same color)

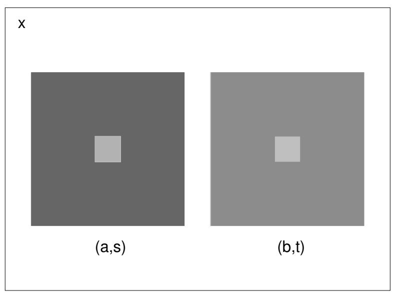
Achromatic color perception

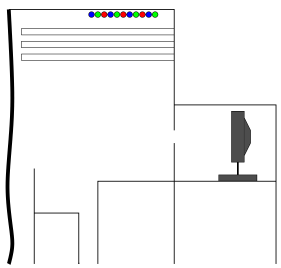
Stimulus configurations

Fechnerian Scaling

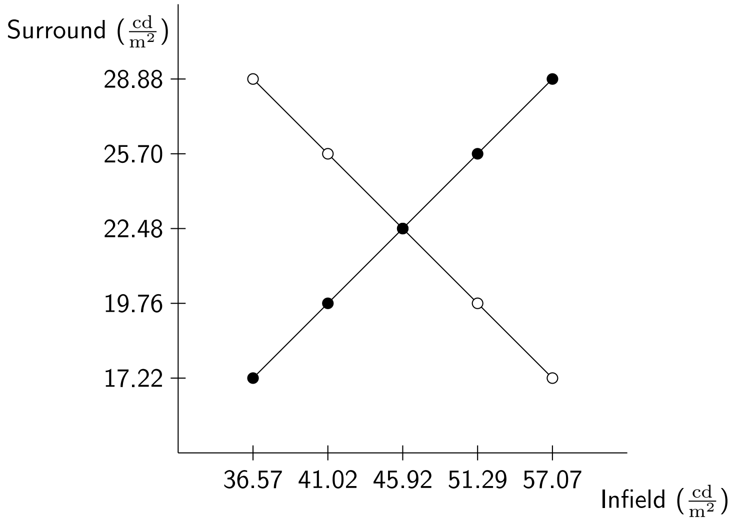
Analysis of data

Stimulus presentation





Stimulus configurations



Stimuli

Stimuli

Achromatic color perception

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Analysis of data

Probability Distance Hypothesis

The **probability-distance hypothesis** states that the probability with which one stimulus is discriminated from another is a function of some subjective distance between these stimuli. (Dzhafarov, 2002, p. 352)

$$\psi(x, y) = f[D(x, y)]$$

Discrimination Probabilities

- Most basic cognitive ability: to tell two stimuli apart from each other
- Fechnerian Scaling computes 'subjective' distances among stimuli from their pairwise discrimination probabilities
- Subjects are required to give one of two answers: *'x and y are the same'* or *'x and y are different'*

$$\psi(x,y) = P(x \text{ and } y \text{ are different})$$

- FS is suitable to describe spaces of arbitrary dimensionality

Subjective Distances

- Subjective distances between stimuli are defined here, measuring the degree of similarity (or dissimilarity) between the underlying representations
- Fechnerian distances satisfy all properties of a metric:

$$D(x,y) \geq 0 \quad \text{non-negativity} \quad (1)$$
$$D(x,y) = 0 \text{ iff } x = y \quad \text{identity of indiscernibles} \quad (2)$$
$$D(x,y) = D(y,x) \quad \text{symmetry} \quad (3)$$
$$D(x,z) \leq D(x,y) + D(y,z) \quad \text{triangle inequality} \quad (4)$$

Regular Minimality

- Most fundamental property of discrimination probabilities
- Only requirement for computation of Fechnerian distances
- For any $x \neq y$

$$\psi(x,x) < min\{\psi(x,y), \psi(y,x)\}.$$

Discrimination probabilities

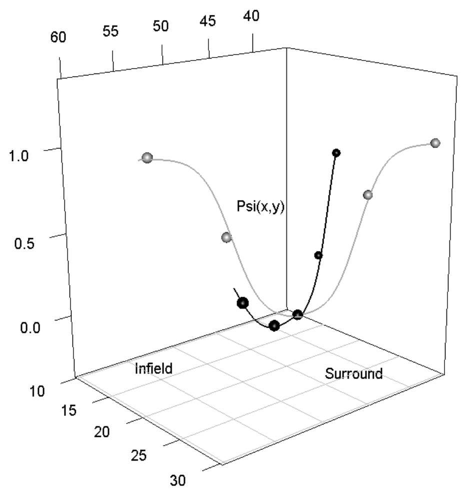
	gray1a	gray2a	gray3a	gray4a	gray5a	gray1b	gray2b	gray4b	gray5b
gray1a	0.00	0.07	0.40	0.76	1.00	0.93	0.33	1.00	1.00
gray2a	0.07	0.01	0.11	0.36	0.91	1.00	0.73	1.00	1.00
gray3a	0.67	0.13	0.01	0.12	0.71	0.98	0.73	0.76	1.00
gray4a	0.91	0.82	0.25	0.01	0.11	1.00	1.00	0.47	0.93
gray5a	1.00	0.97	0.80	0.16	0.01	1.00	1.00	0.47	0.93
gray1b	0.93	1.00	1.00	1.00	1.00	0.00	0.23	1.00	1.00
gray2b	0.33	0.60	0.51	1.00	1.00	0.73	0.00	1.00	0.97
gray4b	1.00	1.00	0.71	0.73	0.53	1.00	1.00	0.01	0.63
gray5b	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.73	0.00

Discrimination probabilities

	gray1a	gray2a	gray3a	gray4a	gray5a	gray1b	gray2b	gray4b	gray5b
gray1a	0.00	0.07	0.40	0.76	1.00	0.93	0.33	1.00	1.00
gray2a	0.07	0.01	0.11	0.36	0.91	1.00	0.73	1.00	1.00
gray3a	0.67	0.13	0.01	0.12	0.71	0.98	0.73	0.76	1.00
gray4a	0.91	0.82	0.25	0.01	0.11	1.00	1.00	0.47	0.93
gray5a	1.00	0.97	0.80	0.16	0.01	1.00	1.00	0.47	0.93
gray1b	0.93	1.00	1.00	1.00	1.00	0.00	0.23	1.00	1.00
gray2b	0.33	0.60	0.51	1.00	1.00	0.73	0.00	1.00	0.97
gray4b	1.00	1.00	0.71	0.73	0.53	1.00	1.00	0.01	0.63
gray5b	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.73	0.00

Psychometric function

'Middle' of cross compared to rest
$\psi(\text{gray3}, y)$



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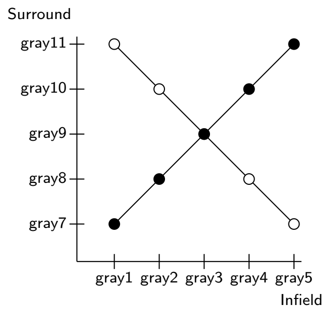

fechner

- Check for regular minimality

```
R> library(fechner)
R> check.regular(psi)
$check
[1] "regular minimality"

$in.canonical.form
[1] TRUE
```

- Calculate Fechnerian Distances

```
R> fs <- fechner(psi, comp=T, check=T)
R> fdis <- fs[[6]]
```

- MDS

```
R> library(smacof)
R> mds2 <- smacofSym(fdis, ndim=2)
```


Fechnerian Distances

	gray1a	gray2a	gray3a	gray4a	gray5a	gray1b	gray2b	gray4b	gray5b
gray1a	0.00	0.12	0.34	0.69	0.94	1.50	0.67	1.61	2.00
gray2a		0.00	0.22	0.57	0.82	1.62	0.79	1.48	1.99
gray3a			0.00	0.35	0.60	1.70	1.01	1.26	1.99
gray4a				0.00	0.25	1.83	1.36	1.12	1.92
gray5a					0.00	1.93	1.61	0.98	1.92
gray1b						0.00	0.97	1.99	2.00
gray2b							0.00	1.99	1.97
gray4b								0.00	1.35
gray5b									0.00

Fechnerian Distances

	gray1a	gray2a	gray3a	gray4a	gray5a	gray1b	gray2b	gray4b	gray5b
gray1a	0.00	0.12	0.34	0.69	0.94	1.50	0.67	1.61	2.00
gray2a		0.00	0.22	0.57	0.82	1.62	0.79	1.48	1.99
gray3a			0.00	0.35	0.60	1.70	1.01	1.26	1.99
gray4a				0.00	0.25	1.83	1.36	1.12	1.92
gray5a					0.00	1.93	1.61	0.98	1.92
gray1b						0.00	0.97	1.99	2.00
gray2b							0.00	1.99	1.97
gray4b								0.00	1.35
gray5b									0.00

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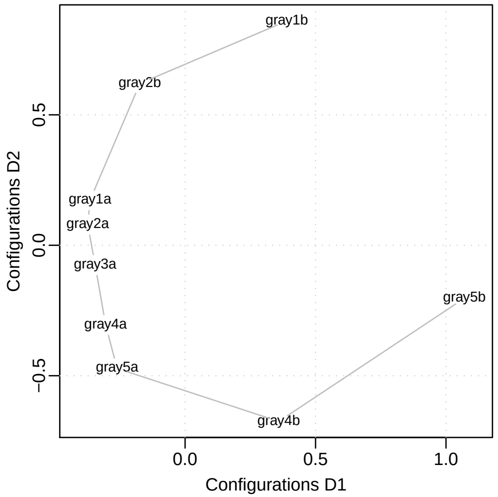
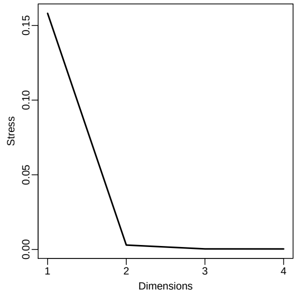
gray3a			0.00	0.35	0.60	1.70	1.01	1.26	1.99
gray4a				0.00	0.25	1.83	1.36	1.12	1.92
gray5a					0.00	1.93	1.61	0.98	1.92
gray1b						0.00	0.97	1.99	2.00
gray2b							0.00	1.99	1.97
gray4b								0.00	1.35
gray5b									0.00

Visual representation of distances





Stimuli

Stimuli

Conclusion

1. This is work in progress!
2. All conclusions are preliminary

Conclusion

- Fechnerian distances of these stimuli can be arranged in a two-dimensional space
- One of the dimensions could be (perceived) lightness of the infield
- Other dimension? Something like "distinguishability"?

Thank you for your attention!

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References

Dzhafarov, E. N. (2002). Multidimensional Fechnerian Scaling: Probability-Distance Hypothesis. *Journal of Mathematical Psychology*, *46*, 352–374.

Gilchrist, A. (2006). *Seeing Black and White*. Oxford: University Press.

Logvinenko, A. D. & Maloney, L. T. (2006). The proximity structure of achromatic surface colors and the impossibility of asymmetric lightness matching. *Perception and Psychophysics*, *68*(1), 76–83.

Niederée, R. (2010). More than three dimensions: What continuity considerations can tell us about perceived color. In J. Cohen & M. Matthen (Eds.), *Color Ontology and Color Science* (pp. 91–122). MIT Press.

Wallach, H. (1948). Brightness Constancy and the Nature of Achromatic Colors. *Journal of Experimental Psychology*, *38*(3), 310.

Additional slides

Fechnerian Scaling

Number of trials for each cell

	gray1a	gray2a	gray3a	gray4a	gray5a	gray1b	gray2b	gray4b	gray5b
gray1a	195	45	60	45	45	15	15	15	15
gray2a	45	105	150	45	45	15	15	15	15
gray3a	60	60	210	60	150	45	45	45	90
gray4a	45	45	60	105	45	15	15	15	15
gray5a	45	135	60	45	105	15	15	15	15
gray1b	15	15	45	15	15	120	30	30	30
gray2b	15	15	90	15	15	30	75	30	30
gray4b	15	15	45	15	15	30	30	75	30
gray5b	15	15	45	15	15	30	75	30	75

Parameters of psychometric functions (logistic model)

$$F(x) = \frac{1}{1 + \exp\left[-(\beta_0 + \beta_1 x)\right]}$$

- Main diagonal:

$$\beta_0^{(1)} = -19.905, \ \beta_1^{(1)} = 0.343$$

$$\beta_0^{(2)} = 24.067, \ \beta_1^{(2)} = -0.617$$

- Secondary diagonal:

$$\beta_0^{(3)} = -46.180, \ \beta_1^{(3)} = 0.901$$

$$\beta_0^{(4)} = 48.000, \ \beta_1^{(4)} = -1.147$$

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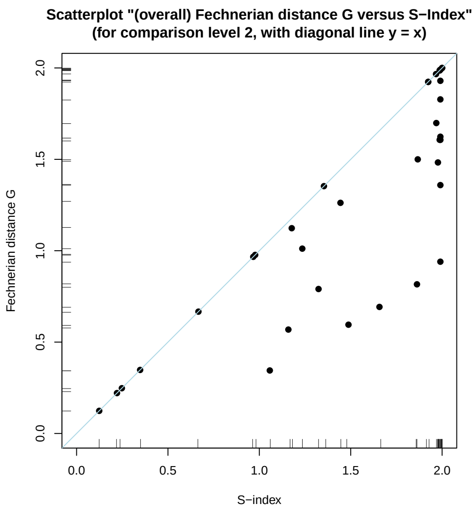
Geodesic loops

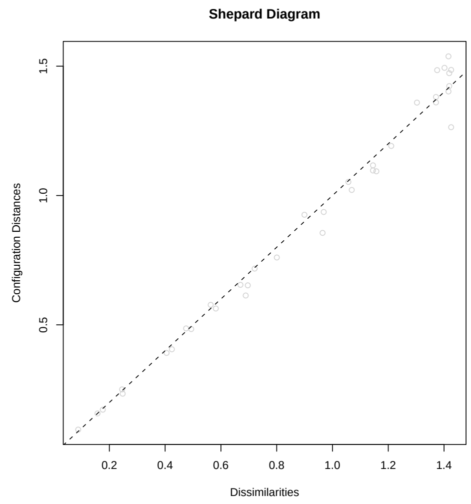
	gray1a	gray2a	gray3a	gray4a	gray5a
gray1a	1a	1a2a1a	1a2a3a2a1a	1a2a3a4a3a2a1a	1a2a3a4a5a4a3a2a1a
gray2a	2a1a2a	2a	2a3a2a	2a3a4a3a2a	2a3a4a5a4a3a2a
gray3a	3a2a1a2a3a	3a2a3a	3a	3a4a3a	3a4a5a4a3a
gray4a	4a3a2a1a2a3a4a	4a3a2a3a4a	4a3a4a	4a	4a5a4a
gray5a	5a4a3a2a1a2a3a4a5a	5a4a3a2a3a4a5a	5a4a3a4a5a	5a4a5a	5a
gray1b	1b2b1a1b	1b2b1a2a1b	1b2b1a2a3a1b	1b2b1a2a3a4a1b	1b2b1a2a3a4a5a1b
gray2b	2b1a2b	2b1a2a1a2b	2b1a2a3a2a1a2b	2b1a2a3a4a3a2a1a2b	2b1a2a3a4a5a4a3a2a1a2b
gray4b	4b3a2a1a2a3a4a4b	4b3a2a3a4a4b	4b3a4a4b	4b5a4a4b	4b5a4b
gray5b	5b1a5b	5b2a5b	5b3a5b	5b4a5b	5b5a5b

	gray1b	gray2b	gray4b	gray5b
gray1a	1a1b2b1a	1a2b1a	1a2a3a4a4b3a2a1a	1a5b1a
gray2a	2a1b2b1a2a	2a1a2b1a2a	2a3a4a4b3a2a	2a5b2a
gray3a	3a1b2b1a2a3a	3a2a1a2b1a2a3a	3a4a4b3a	3a5b3a
gray4a	4a1b2b1a2a3a4a	4a3a2a1a2b1a2a3a4a	4a4b5a4a	4a5b4a
gray5a	5a1b2b1a2a3a4a5a	5a4a3a2a1a2b1a2a3a4a5a	5a4b5a	5a5b5a
gray1b	1b	1b2b1b	1b4b1b	1b5b1b
gray2b	2b1b2b	2b	2b4b2b	2b5b2b
gray4b	4b1b4b	4b2b4b	4b	4b5b4b
gray5b	5b1b5b	5b2b5b	5b4b5b	5b

S-Index



Diagnostic plots MDS



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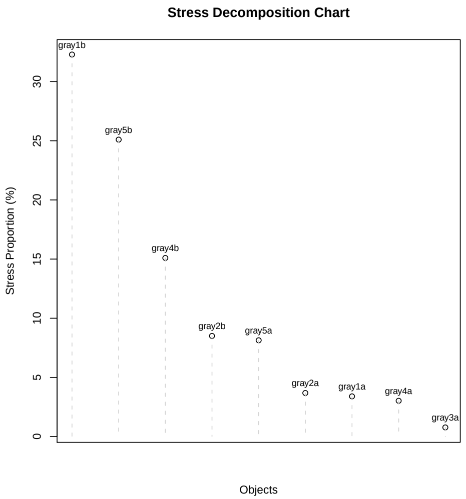

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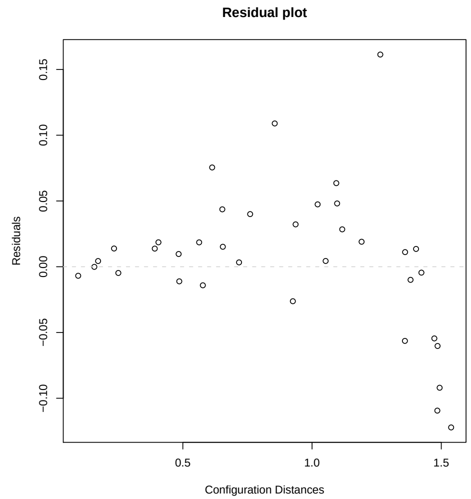

Additional slides

Fechnerian Scaling

Regular Minimality

- Most fundamental property of discrimination probabilities
- Only requirement for computation of Fechnerian distances
- For any $x \neq y$

$$\psi(x, x) < min\{\psi(x, y), \psi(y, x)\}.$$

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- Example for discrete object set

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- Example for discrete object set

	y_1	y_2	y_3	y_4
x_1	0.5	0.7	1.0	1.0
x_2	1.0	0.5	1.0	0.6
x_3	0.9	0.9	0.8	0.1
x_4	0.6	0.6	0.1	0.8

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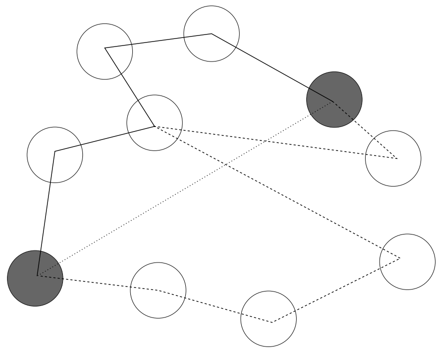
Psychometric Increments

- We define psychometric increments for each observation area

$$\phi^{(1)} = \psi(x,y) - \psi(x,x)$$
$$\phi^{(2)} = \psi(y,x) - \psi(x,x)$$

- Due to regular minimality all psychometric increments are positive
- Minima $\psi(x,x)$ can have different values (nonconstant self-dissimilarity)

Discrete Object Space



In a discrete space Fechnerian computations are performed by taking sums of psychometric increments for all possible chains leading from one point to another (3 examples shown here).

Oriented Fechnerian Distance

- Consider a chain from s_i to s_j, with $k \geq 2$
- Psychometric length of the first kind

$$L^{(1)}(x_1, x_2, ..., x_k) = \sum_{m=1}^{k} \phi^{(1)}(x_m, x_{m+1})$$

- Finite number of psychometric lengths across all possible chains connecting s_i and s_j
- Oriented Fechnerian distance:

$$G_1(s_i, s_j) = L^{(1)}_{min}(s_i, s_j)$$

- Satisfies all properties of a metric except symmetry
- Oriented distances are not computed across observation areas but rather within observation areas

Fechnerian Distance

- For better interpretation we add up all oriented Fechnerian distances from s_i to s_j and from s_j to s_i
- Overall Fechnerian distance

$$G(s_i, s_j) = G_1(s_i, s_j) + G_1(s_j, s_i) = G_2(s_i, s_j) + G_2(s_j, s_i)$$

- Satisfies all properties of a metric
- Does not depend on observation area
- Gives us a readily interpretable measure of the ‘subjective’ distance between s_i and s_j