Mokken Scale Analysis

in MSP, in R, in SPSS

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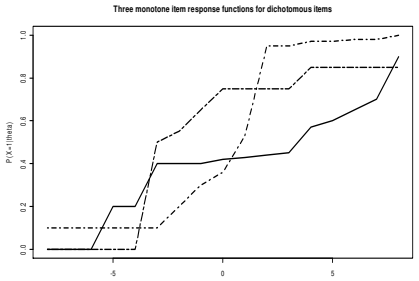
Monotone Homogeneity Model (MHM)

- Notation: $X_1, \ldots, X_j, \ldots, X_J$: item scores; θ: latent trait
- MHM (Mokken, 1971): General IRT model for $P(X_j = x|\theta)$

  Alternative names: Unidimensional Latent Variable Model (e.g., Holland & Rosenbaum, 1991)
  Nonparametric Graded Response Model (e.g., Hemker et al., 1997),

Assumptions:

1. Unidimensionality
2. Local independence
3. Monotonicity: $P(X_j \geq x|\theta)$ nondecreasing in θ



Monotone Homogeneity Model (MHM)

- Goodness of fit investigated using *observable consequences* (e.g., Mokken, 1971, Sijtsma & Molenaar, 2002, Sijtsma & Junker, 2000, Holland & Rosenbaum, 1990, Rosenbaum, 1984)

  E.g.: MHM => $\text{Cov}(X_i, X_j) \geq 0$

- Property I: All well-known unidimensional IRT models are a special case of the MHM (Hemker et al., 2001, Van der Ark, 2001) :

  E.g., Rasch model, 2PLM, 3PLM, GRM, PCM, gPCM

- Property II: MHM implies stochastic ordering of θ by X_+ (e.g., Grayson, 1988, Hemker et al., 1996, Van der Ark, 2005, Van der Ark & Bergsma, 2010).

  E.g.: $E(\theta|X_+ = 12) \geq E(\theta|X_+ = 11)$

Mokken Scale Analysis

Scaling procedure for dichotomous and polytomous items.

(e.g., Mokken, 1971; Sijtsma & Molenaar, 2002; Van der Ark, 2007)

(# citations in Google Scholar: "graded response model" 1,690; "Mokken" 3,450; "Rasch model" 12,000; "factor analysis" 1,600,000)

1. Automated item selection procedure (AISP)

   Partitions a set of items into *Mokken scales* (possibly leaving some items unscalable)

2. Several methods to check observable properties of the MHM

   (and other nonparametric IRT models; e.g. check of nonintersection of item response functions)

Mokken Scale Analysis in MSP

- **Mokken Scaling for polytomous items** (MSP, Molenaar & Sijtsma, 2000)
  - User-friendly!!
  - Commercial package (€225 one-user licence)
  - DOS program with Windows shell (apparently fails under Windows 7)
  - Difficult to add new features

Mokken Scale Analysis in R

- **R package `mokken`** (Van der Ark, 2007, 2010)
  - Not so user-friendly because typical users of Mokken scale analysis do not use R.
  - Freeware
  - Easy to add new features

```
library(mokken)
data(acl)
communality <- acl[,1:10]
communality[1:3,]
```

```
     reliable honest unscrupulous* deceitful* unintelligent* obnoxious* thankless* unfriendly* dependable cruel*
[1,]        3      3            2          4              4          4          4           4          3      4
[2,]        2      4            4          3              3          4          1           3          4      4
[3,]        2      3            3          3              3          3          4           3          3      4
```

Mokken Scale Analysis in R
Automated Item Selection Procedure

```
scale <- aisp(communality, search = "normal")
scale
```

```
reliable           1
honest             1
unscrupulous*      0
deceitful*         1
unintelligent*     0
obnoxious*         2
thankless*         2
unfriendly*        2
dependable         1
cruel*             2
```

Mokken Scale Analysis in R
Scalability Coefficients

```
H1 <- coefH(communality[,scale==1])
names(H1)
round(H1$Hij,2)
reliable honest deceitful* dependable
reliable       1.00   0.53        0.33        0.72
honest         0.53   1.00        0.28        0.55
deceitful*     0.33   0.28        1.00        0.32
dependable     0.72   0.55        0.32        1.00
round(H1$Hi,2)
 reliable     honest deceitful* dependable
      0.50       0.43       0.31       0.50
round(H1$H,2)
0.43
```

Mokken Scale Analysis in R
Check of model assumptions e.a.

```
check.monotonicity
check.iio
check.restscore
check.reliability
check.pmatrix
```

S3 –methods available for `summary()` and `plot()`. Example

```
> M1 <- check.monotonicity(communality[,scale==1])
> summary(M1)
> plot(M1)
> check.reliability(communality[,scale==1],LCRC=TRUE)
```

Mokken Scale Analysis in R
New features

- Automated item selection procedure using genetic algorithm (Straat et al., 2010)
- Investigating invariant item orderings (popular in clinical nursing) (Ligtvoet et al., 2010, 2011)
- New reliability coefficients (Van der Ark et al., 2011)
- Standard errors for scalability coefficients (future) (Van der Ark et al., 2008; Kuijpers et al., 2011)

Having software available increases the chance of publication

Mokken Scale Analysis in SPSS
(i.e. use R code in SPSS)

```
BEGIN PROGRAM R.
casedata <- spssdata.GetDataFromSPSS(variables
  =c("v_21, v_20, v_23, v_25, v_19, v_24, v_22"))
library("mokken")

print( "Scalability Coefficients" )
coefH(casedata)

print( "Monoticity in Mokken Scale Analysis" )
MonoScale <- summary(check.monotonicity(casedata))
print(MonoScale)
spsspivottable.Display(MonoScale, title="Results
  Monotonicity",
format=formatSpec.GeneralStat)
END PROGRAM R.
```

Mokken Scale Analysis in SPSS
(i.e. use R code in SPSS)

- Even better?: R code in the SPSS pull-down menu
- Frustrating
  - Difficult programming
  - Requires huge add-on files
  - Requires close inspection of computers (access rights)
- But,
  - Improving (SPSS 18)
  - Much larger audience

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