Residual Information for Absolute Model Fit

Feb 2020

Charles Driver

Max Planck Institute for Human Development

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- I hope we can improve on this by approaching model fit differently.

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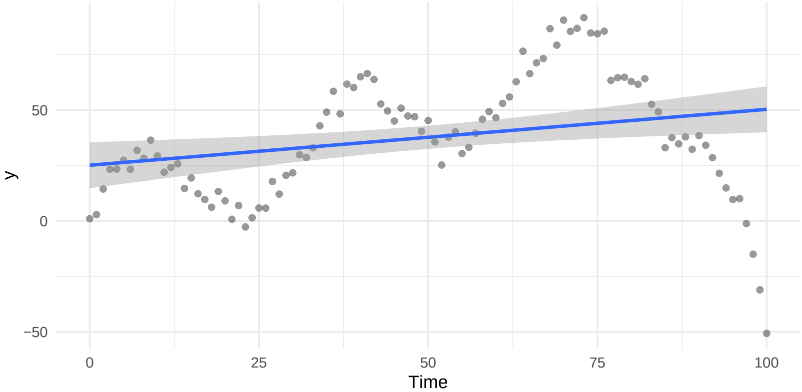
- Straightforward with simple, deterministic systems and yes / no questions – prediction errors that can't be explained by instrumentation imply the model is inadequate.

- Not so straightforward when we know in advance the model is imperfect.



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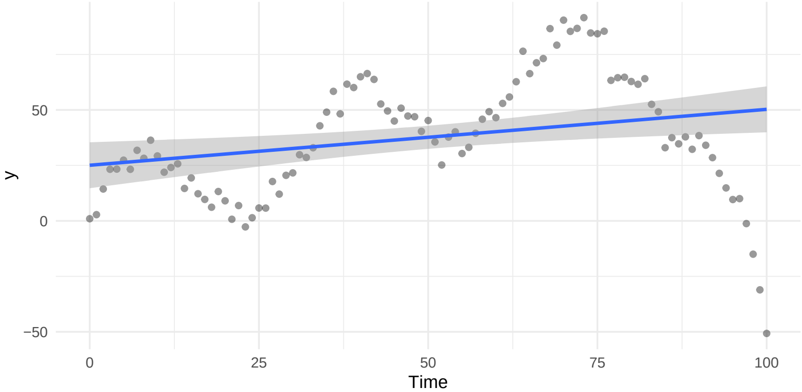

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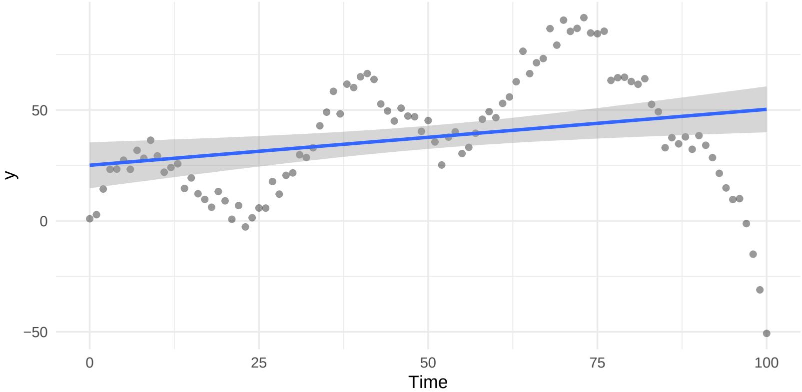

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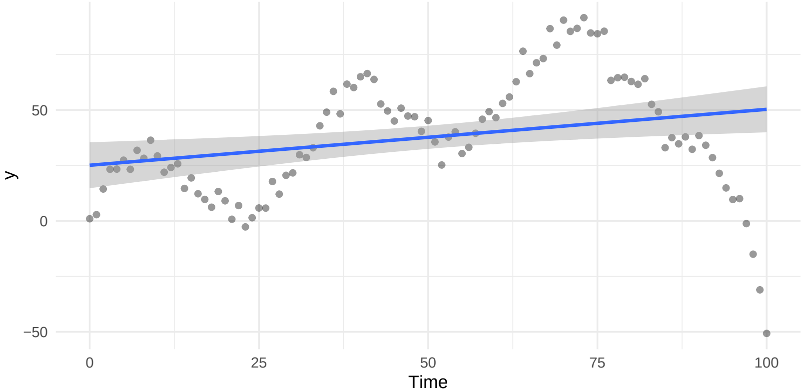

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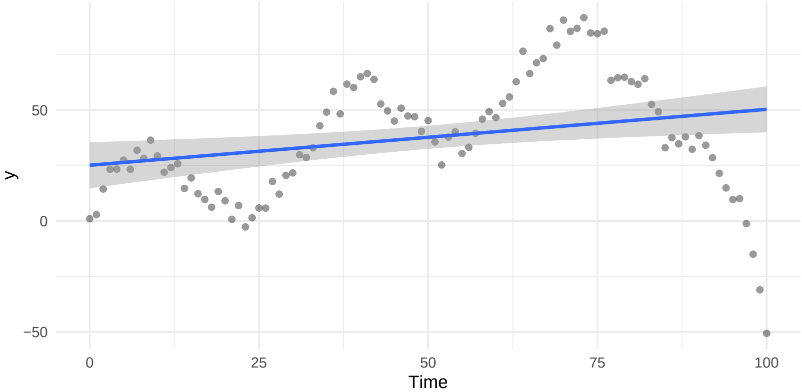

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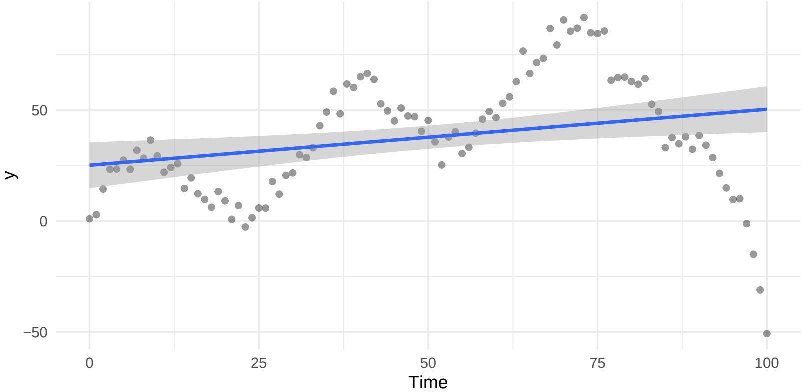
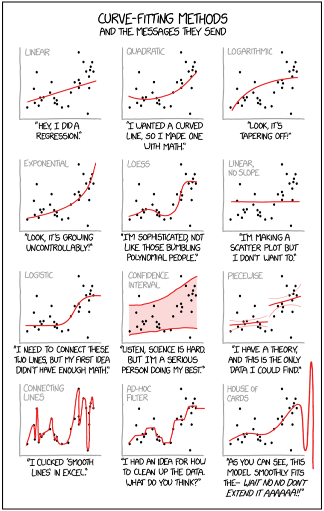
    - Interpreted loosely, it leads to crazy inferences.
    - Uncertainty about the parameter tells us nothing about the ‘trueness’ of the parameter.



How to quantify model fit then?

- Lots of technical concerns, but at base:



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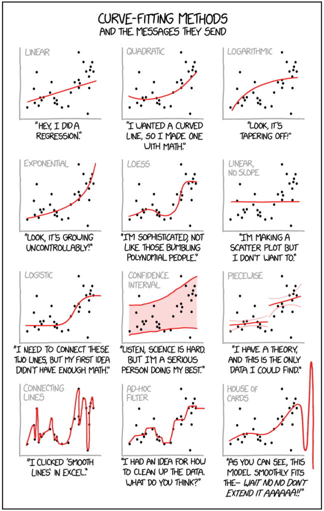
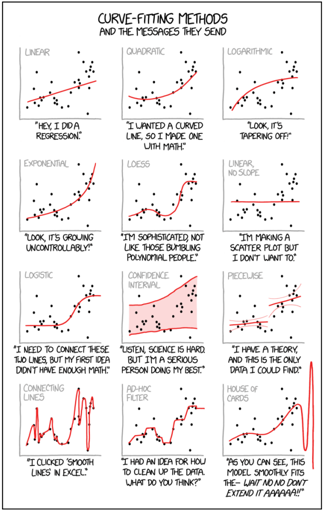


- Lots of technical concerns, but at base:
  - How well does the fitted model predict new data?
  - How well do alternative models predict new data?



What should the alternative models be?

- Theory based:

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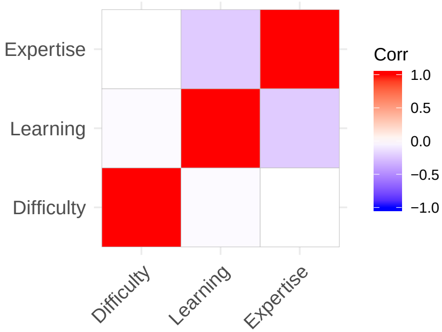
- Data based:
  - Arbitrary 'standard' model – e.g. saturated covariance matrix.

- Take some structuring of data.



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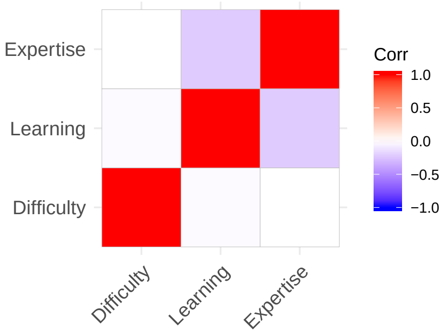

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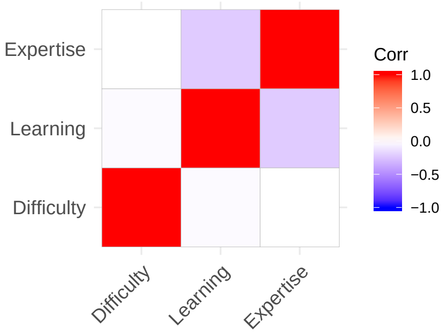

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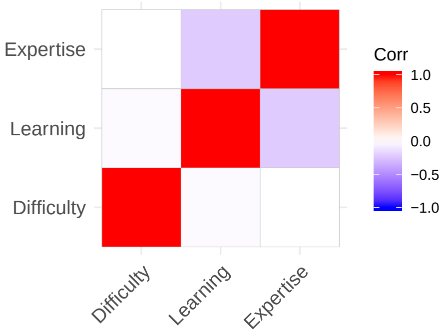

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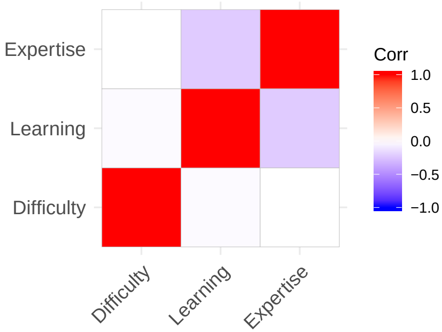
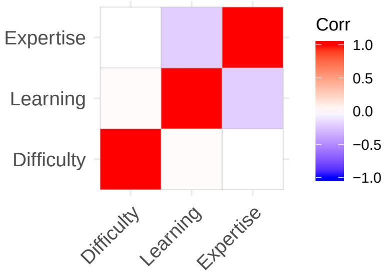
- Estimate covariance matrix.
- Compare saturated model with our model – obtain estimate of distance from saturated model.
- If the true model can be represented by some covariance matrix of our data structure, we have estimate of distance from true model.
- Otherwise, we just have estimate of distance from best linear model given our arbitrary data structuring.




Expertise
Learning
Difficulty
Difficulty
Learning
Expertise
Corr
1.0
0.5
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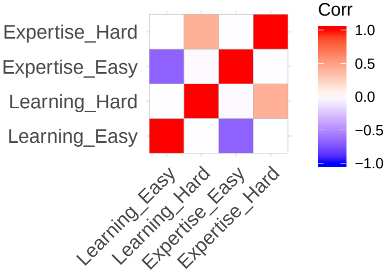
−0.5
−1.0


Expertise_Hard
Expertise_Easy
Learning_Hard
Learning_Easy
Learning_Easy
Learning_Hard
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Expertise_Hard
Corr
1.0
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0.0
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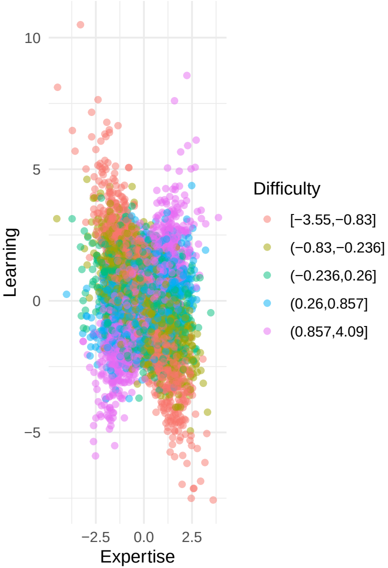
10
5
0
−5
Learning
−2.5
0.0
2.5
Expertise
Difficulty
[−3.55,−0.83]
(−0.83,−0.236]
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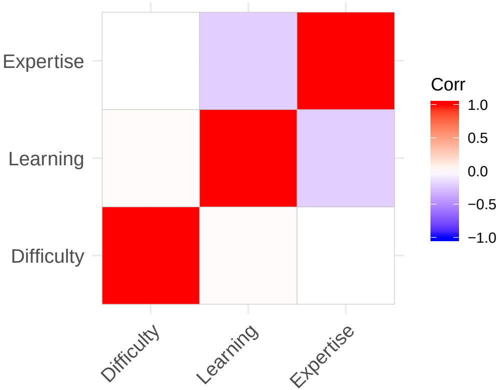
(0.26,0.857]
(0.857,4.09]

- Easy to obtain ‘perfect’ fit wrt saturated covariance.



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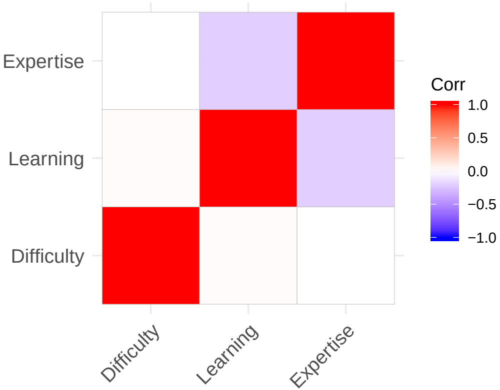

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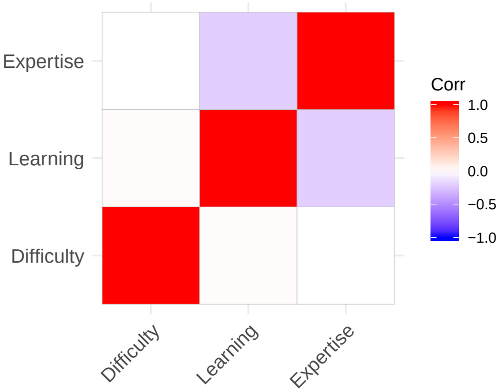
- Can we do better?



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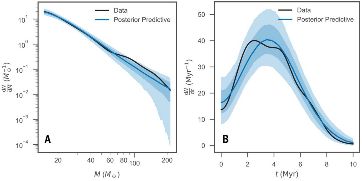

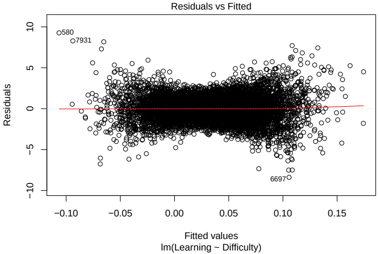

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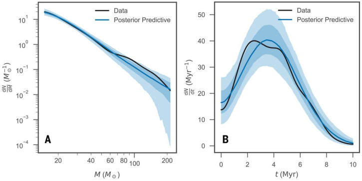

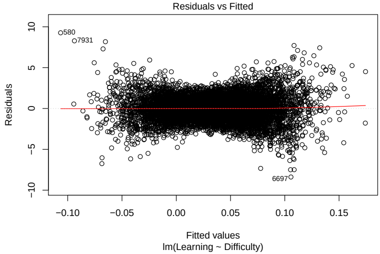

- Posterior predictive checks:
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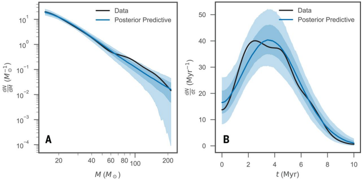

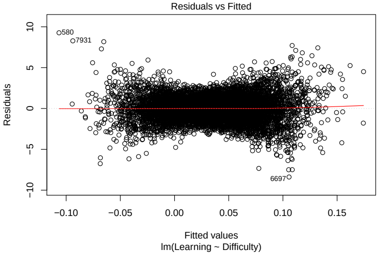

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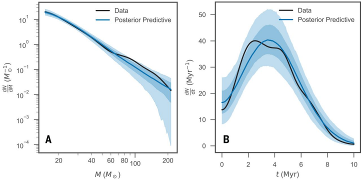

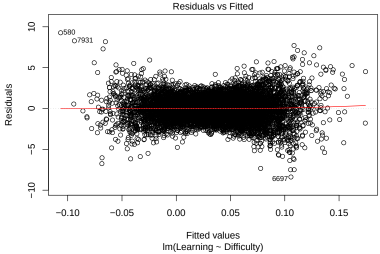

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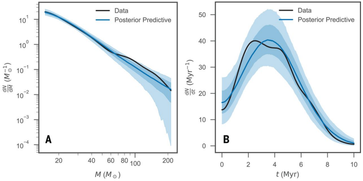

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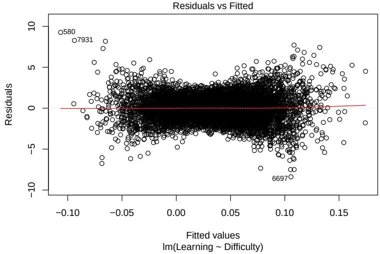

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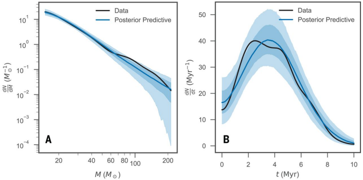

- Residual checks:
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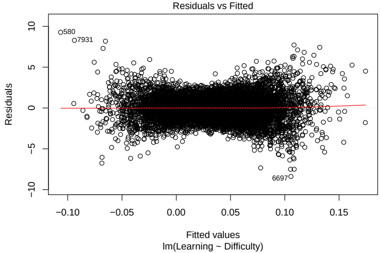

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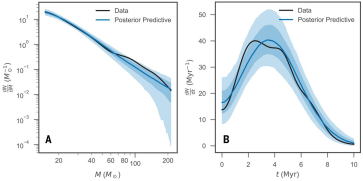
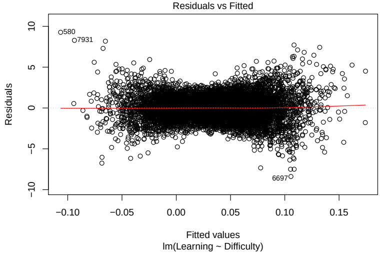
  - Visually check residuals from fit for patterns, some specific tests.
  - Useful, good guide to improving model, but: non-general, too much of an art, typically univariate.





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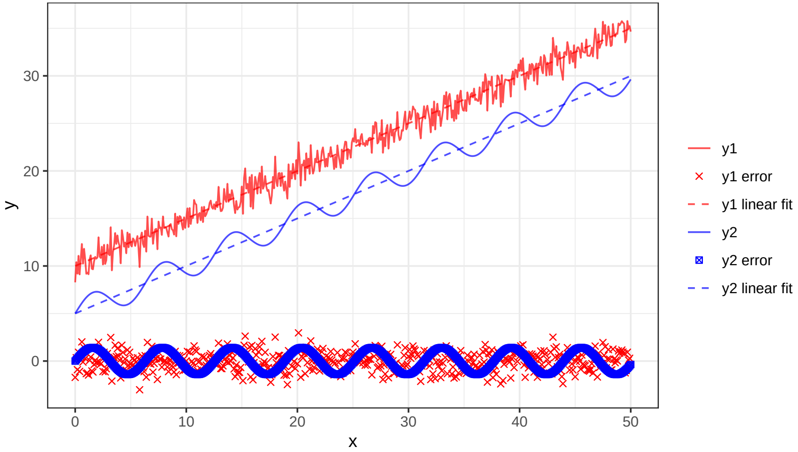

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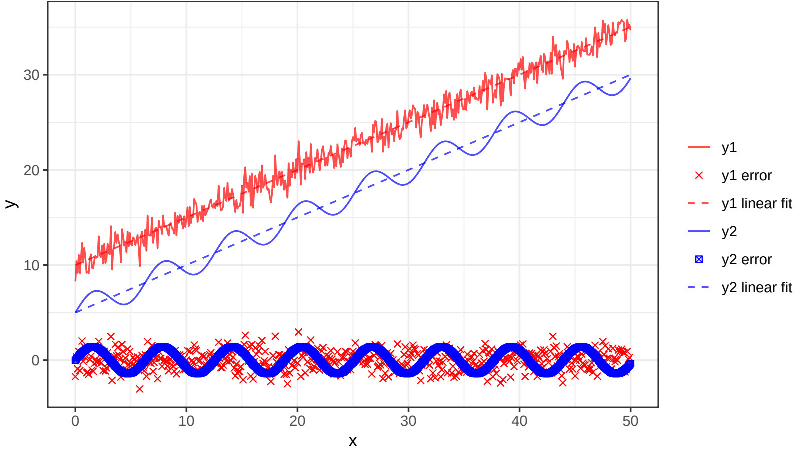
- Therefore, information that our residuals do contain, can be used to quantify distance from best possible model.



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- Need some general function to detect and quantify structure in data.

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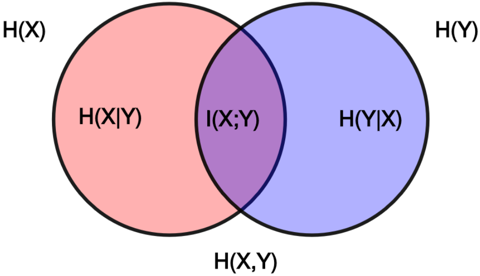

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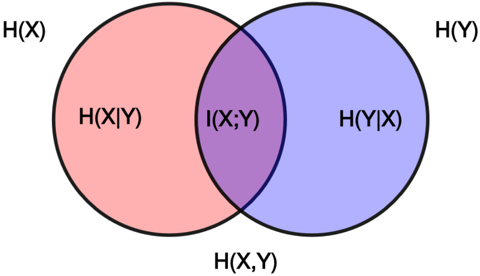

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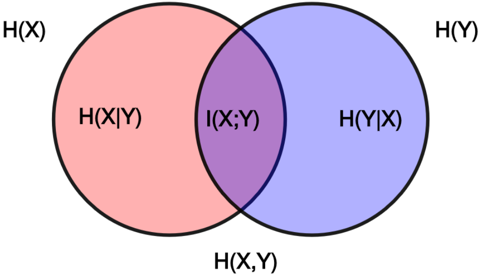
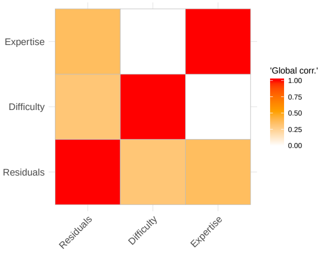
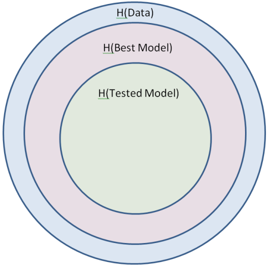
- Conditional entropy, $H(X|Y) = H(X) - I(X,Y)$, quantifies information unique to X with respect to Y.



- Mutual information (nonlinear covariance) between residuals, expectations, and any additional covariates.





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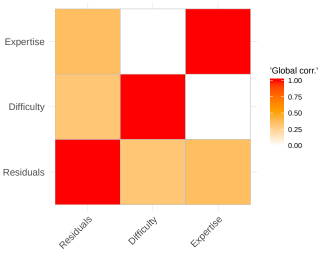

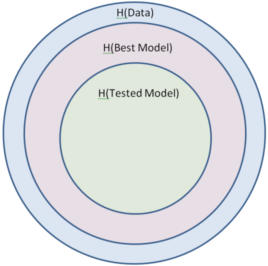

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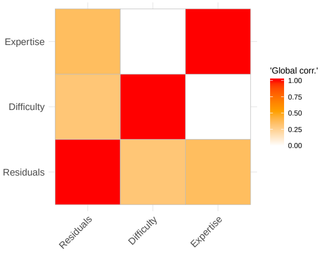

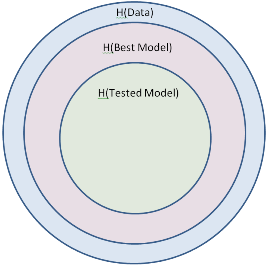

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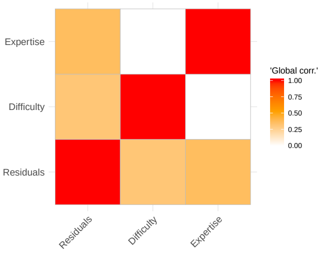
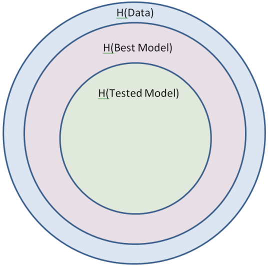
  - Where there is shared information between data and residuals – model can be improved.





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  - We can base estimates from the residuals instead, leveraging the initial model.
  - Relative differences are sufficient to provide some guide as to goodness of fit.
- Overfitting can be handled either by overfitting baseline estimates in a similar fashion, or cross validation.

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- Current plan: R software to provide fit / diagnostics for arbitrary statistical model.

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    - Helps bridge some of the performance divide between prediction oriented and explanation oriented approaches.