The semnova Package for Latent Repeated Measures ANOVA

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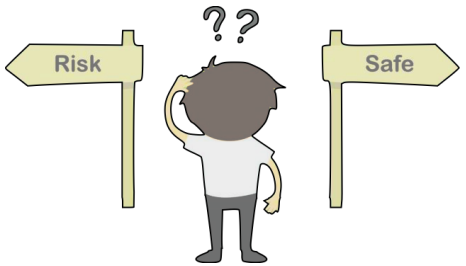
RWTH AACHEN UNIVERSITY

Exemplary Research Question

Does noise affect risky decision making? (Syndicus et al., 2016)

no noise vs. speech vs. office noises



Measured variables

- The Choice Dilemma Questionnaire (12 items, percentages)
- The Risk Scenario Questionnaire (20 items, 10-point scale)

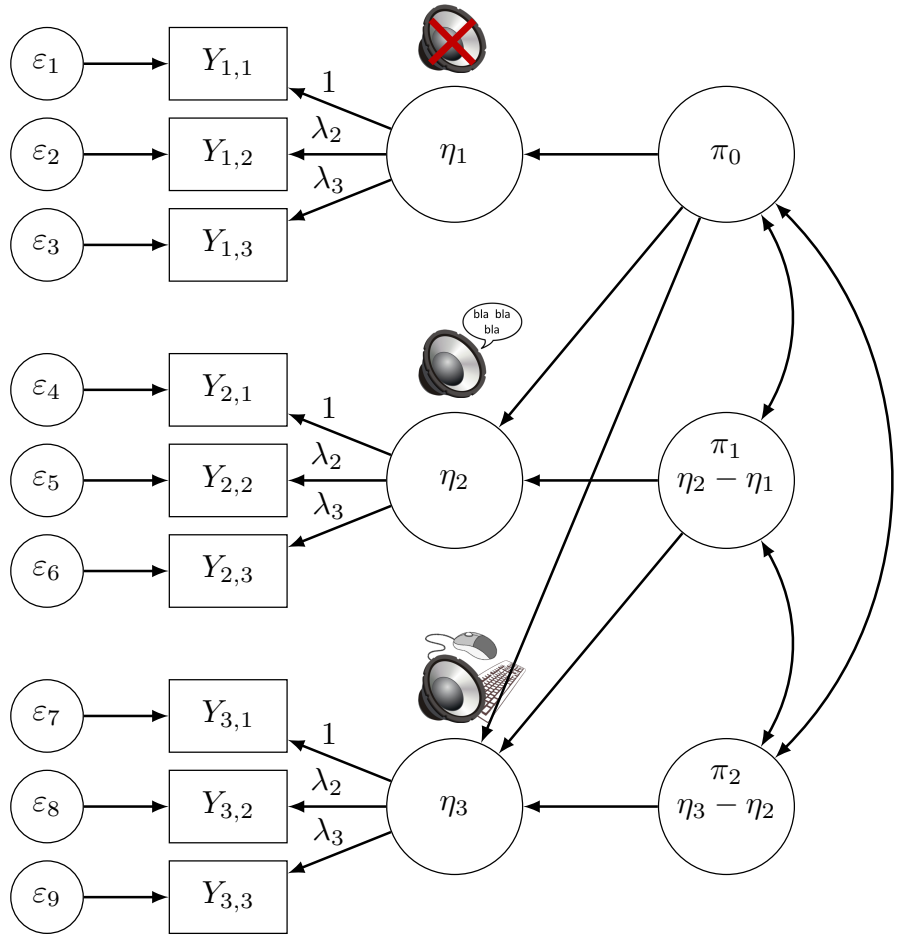
$\Rightarrow$ traditionally analyzed by repeated measures ANOVA (GLM) using averaged indicators (mean scores) as dependent variables or using separate analyses

Advantages of SEM over repeated measures ANOVA

- More power due to explicit error modeling
- More complex covariance structures allowed
  - Data does not have to satisfy sphericity
  - Covariance structure may differ among groups
  - Test for error structures available (e.g. compound symmetry, sphericity)
- Interindividual differences may be investigated
  - Exogenous variables may be included explaining differences among conditions
- Advanced methods of handling missing data and violations of normality available
- Model fit available
- Robust estimators available
- Test for measurement invariance

A Minimal Example

Latent repeated measures ANOVA is based on the latent growth components approach (Mayer et al., 2012)

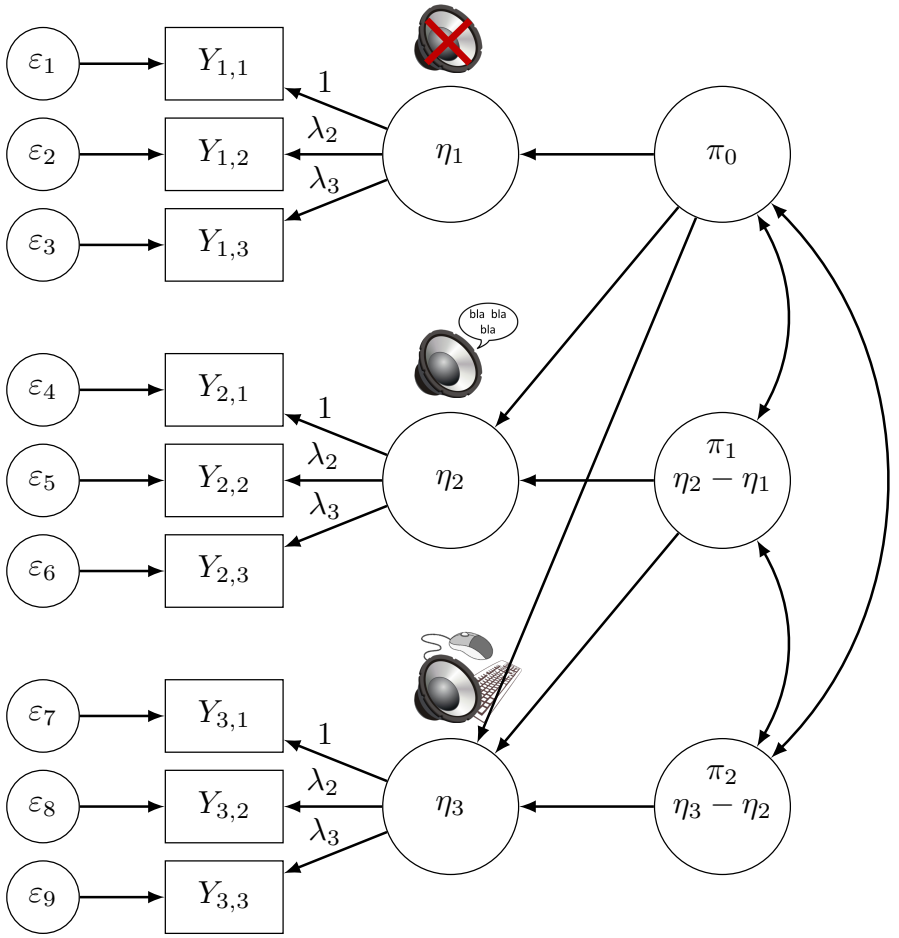


A Minimal Example

Latent repeated measures ANOVA is based on the latent growth components approach (Mayer et al., 2012)

In general, transform η into latent effect variables π:

$$\boldsymbol{\pi} = \overset{\mathbf{C}}{\begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}} \boldsymbol{\eta}$$



A Minimal Example

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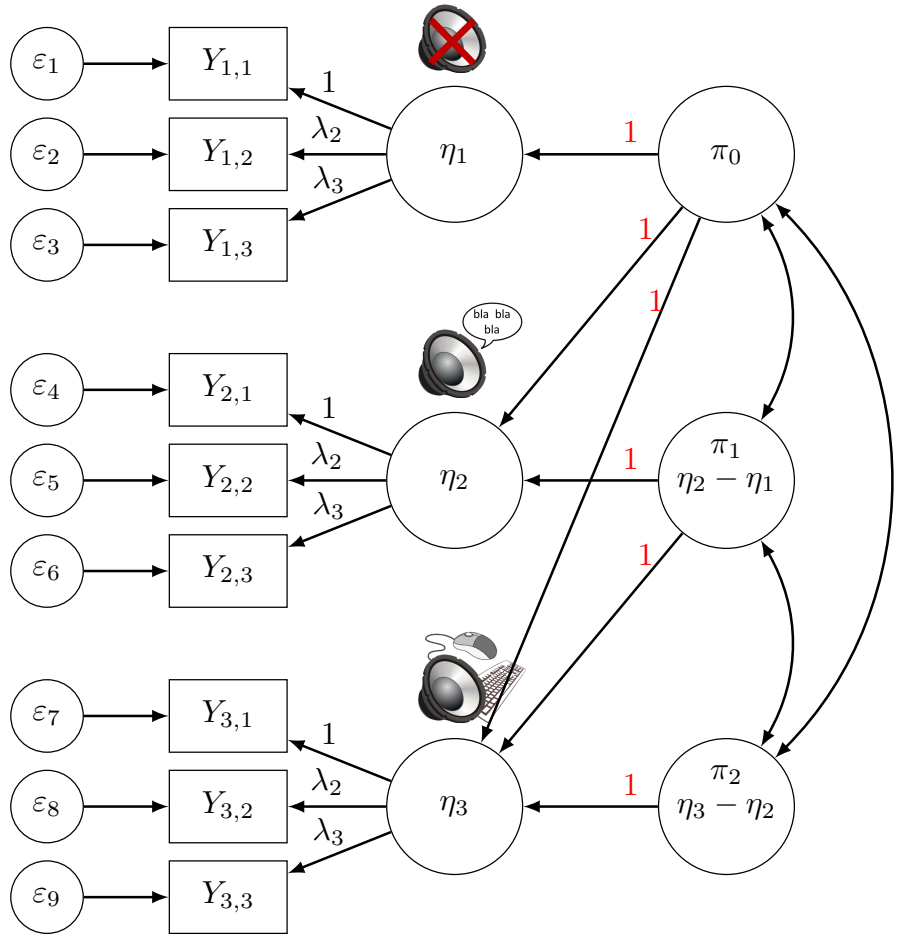
In general, transform η into latent effect variables π:

$$\boldsymbol{\pi} = \overset{\mathbf{C}}{\begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}} \boldsymbol{\eta}$$

Add row to make C invertible:

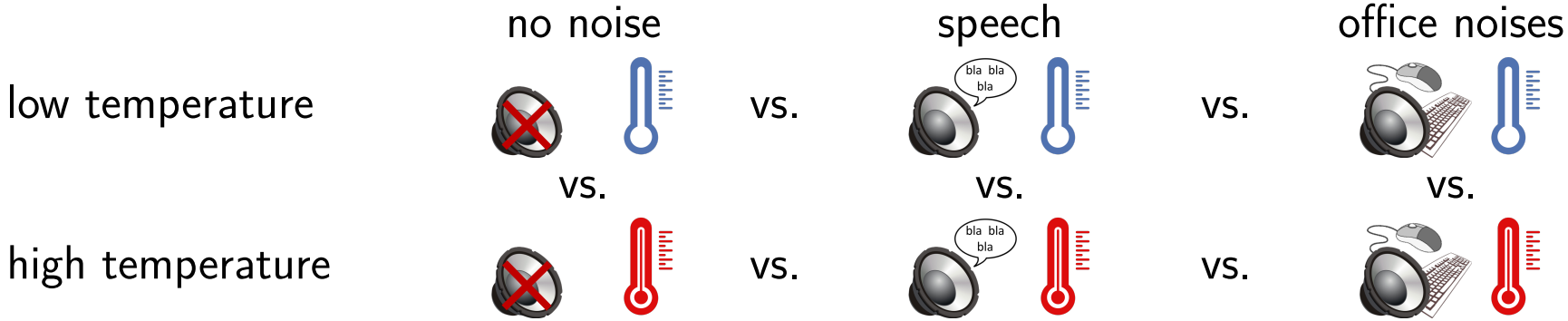
$$\boldsymbol{\pi} = \overset{\mathbf{C}}{\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}} \boldsymbol{\eta} \quad \Leftrightarrow \quad \boldsymbol{\eta} = \overset{\mathbf{B}^*}{\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}} \boldsymbol{\pi}$$

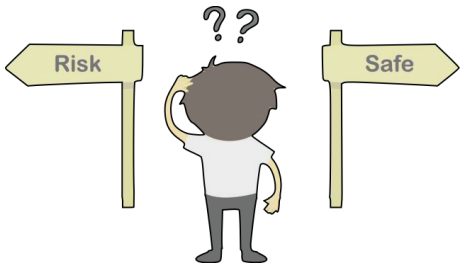


The semnova Package for Latent Repeated Measures ANOVA — Benedikt Langenberg (langenberg@psych.rwth-aachen.de); Axel Mayer — RWTH Aachen University — 28.02.2020

A Larger Example

Do noise or temperature affect risky decision making? (Syndicus et al., 2016)





Measured variables (again)

- The Choice Dilemma Questionnaire (12 items, percentages)
- The Risk Scenario Questionnaire (20 items, 10-point scale)

A Larger Example

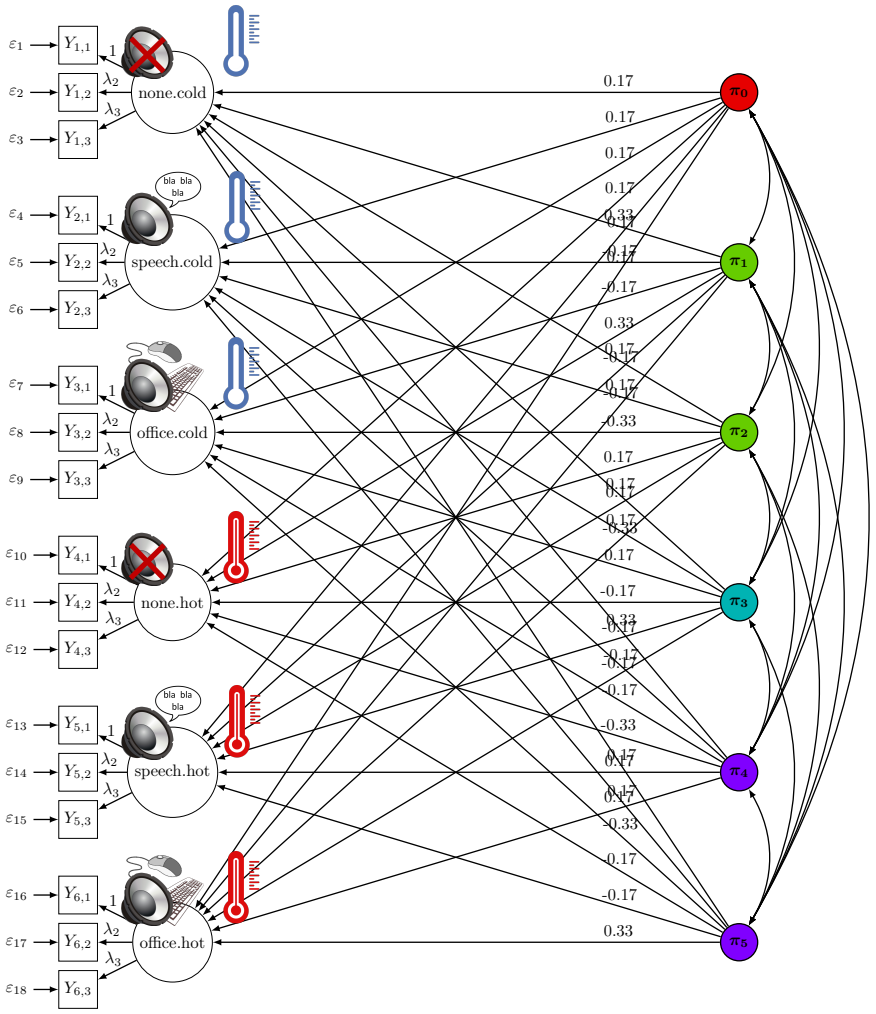
What the package does:

$$\mathbf{C} = \begin{array}{ll} & \text{Factor: noise / temp.} \\ & \begin{array}{cccccc} \text{none} & \text{low} & \text{high} & \text{none} & \text{low} & \text{high} \\ \text{cold} & \text{cold} & \text{cold} & \text{hot} & \text{hot} & \text{hot} \end{array} \\ \text{contrasts} \begin{array}{l} \text{intercept}\,(\pi_0) \\ \text{noise1}\,(\pi_1) \\ \text{noise2}\,(\pi_2) \\ \text{temp1}\,(\pi_3) \\ \text{noise1:temp1}\,(\pi_4) \\ \text{noise2:temp1}\,(\pi_5) \end{array} & \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 1 & -1 \\ 1 & 1 & 1 & -1 & -1 & -1 \\ 1 & -1 & 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 & -1 & 1 \end{pmatrix} \end{array}$$

dependent variables

$$\mathbf{B}^* = \begin{array}{ll} & \text{Factor: noise / temp.} \\ & \begin{array}{cc} \text{none} & \text{cold} \\ \text{low} & \text{cold} \\ \text{high} & \text{cold} \\ \text{none} & \text{hot} \\ \text{low} & \text{hot} \\ \text{high} & \text{hot} \end{array} \\ \text{dep. variables} & \begin{pmatrix} 0.17 & 0.33 & 0.17 & 0.17 & 0.33 & 0.17 \\ 0.17 & -0.17 & 0.17 & 0.17 & -0.17 & 0.17 \\ 0.17 & -0.17 & -0.33 & 0.17 & -0.17 & -0.33 \\ 0.17 & 0.33 & 0.17 & -0.17 & -0.33 & -0.17 \\ 0.17 & -0.17 & 0.17 & -0.17 & 0.17 & -0.17 \\ 0.17 & -0.17 & -0.33 & -0.17 & 0.17 & 0.33 \end{pmatrix} \end{array}$$

contrasts: (π_0) intercept, (π_1) noise1, (π_2) noise2, (π_3) temp1, (π_4) noise1:temp1, (π_5) noise2:tmp1



A Larger Example

R package **semnova** implements latent repeated measures ANOVA using the SEM software package **lavaan** (Rosseel, 2012)

`semnova(...)`

- `data`: Data frame.
- `idata`: Matrix. Design matrix of the within-subject factors. Similar to the `idata` object in the `car` package.
- `mmodel`: List of character vectors. Each Element represents a latent dependent variable measured by the manifest indicators that are included in the character vector.
- (`icontrasts`: Character string. Default is "`contr.sum`". Specifies the type of contrasts to be used.)

A Larger Example

R package **semnova** implements latent repeated measures ANOVA using the SEM software package **lavaan** (Rosseel, 2012)

```
> head(data)

          Y11        Y12          Y13        Y21        Y22         Y23        Y31        Y32         Y33
1  0.2004584  1.7122335  1.86085243 -0.2483409  1.6641324  1.13194771 -0.1965220 -0.3876609  0.72844499
2  0.1862405  0.2657459 -0.09357174 -0.4528128 -0.1170475  1.19826603 -0.9518866 -0.9960841  2.29458413
3  4.2185414  4.1228080  0.72206631  1.5574117  0.2289177 -0.04011789  2.9190039  3.1094043  1.00288043
4  1.4312455  1.7345077  1.13627636  0.3325998  0.9038465  2.10896642  1.6668742  1.4398952  0.74878589
5  2.1724362  1.6230909  1.01891961  0.1978093 -0.6514590  0.80023643  0.2205186  2.4143326 -0.08174437
6  1.6229890  2.5948945 -0.01458020  3.0525912  1.7065496  1.38144415  3.6329593  2.2300305  1.78360290
          Y41        Y42        Y43        Y51        Y52         Y53        Y61        Y62        Y63
1 2.28418999  0.7364414  1.1718701  0.4309800  2.1110208 -0.04430411  1.0015881 -0.2578211  0.5504424
2 0.04583038 -0.4760048  0.8953298  0.1435606  0.9644196 -0.74461258  0.3374447  3.1675914  1.4721956
3 2.04458084  1.1012540  3.6971539  3.7982794  1.1863811  3.71389785  3.0867334  1.0604590  0.9689124
4 0.90092458  0.5537761  1.4479135  0.6998906  1.4130335  1.26029682  1.2081589  0.2769748 -0.9719528
5 1.94201956  1.7937876  2.1433103  0.1461332 -0.5443832  1.30563461  1.0690851  0.2793267  1.9604143
6 1.69692936  1.4636682  0.5518675  3.4503364  0.2924008  2.18199691  2.6190934  1.3106907  1.8708039
```

A Larger Example

R package **semnova** implements latent repeated measures ANOVA using the SEM software package **lavaan** (Rosseel, 2012)

```
> library(semnova)

> fit <- semnova(
      data   = data,
      idata  = expand.grid(
          noise       = c("none", "speech", "office"),
          temperature = c("cold", "hot")
      ),
      mmodel = list(
          none.cold = c("Y11", "Y12", "Y13"),
          low.cold  = c("Y21", "Y22", "Y23"),
          high.cold = c("Y31", "Y32", "Y33"),
          none.hot  = c("Y41", "Y42", "Y43"),
          low.hot   = c("Y51", "Y52", "Y53"),
          high.hot  = c("Y61", "Y62", "Y63")
      ),
  )

> summary(fit)

```

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      ),
  )

> summary(fit)

```

A Larger Example

Output:

```
------------------------------------------

term: noise

Response transformation matrix:
            noise1   noise2
none.cold     -0.5  0.28868
speech.cold    0.0 -0.57735
office.cold    0.5  0.28868
none.hot      -0.5  0.28868
speech.hot     0.0 -0.57735
office.hot     0.5  0.28868

multiv. tests:
      Df test stat approx F num Df den Df  Pr(>F)
Wald   2    3.8139   1.9069      2    198 0.15125
Wilks  1    0.9047   5.1618      2     98 0.00739 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

univ. test:
       Sum Sq num Df Error SS den Df F value   Pr(>F)
F-test 3.8361      2   71.606    198  5.3037 0.005705 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

------------------------------------------
```

```
------------------------------------------

term: temperature

Response transformation matrix:
            temperature1
none.cold        -0.40825
speech.cold      -0.40825
office.cold      -0.40825
none.hot          0.40825
speech.hot        0.40825
office.hot        0.40825

multiv. tests:
      Df test stat  approx F num Df den Df Pr(>F)
Wald   1   0.00263 0.0026270      1     99 0.9592
Wilks  1   0.99995 0.0052905      1     99 0.9422

univ. test:
          Sum Sq num Df Error SS den Df F value Pr(>F)
F-test 0.0029077      1   54.412     99  0.0053 0.9422

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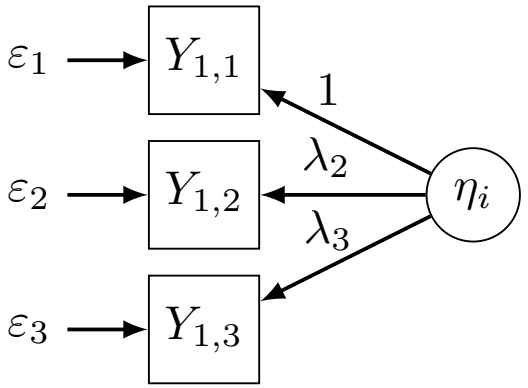
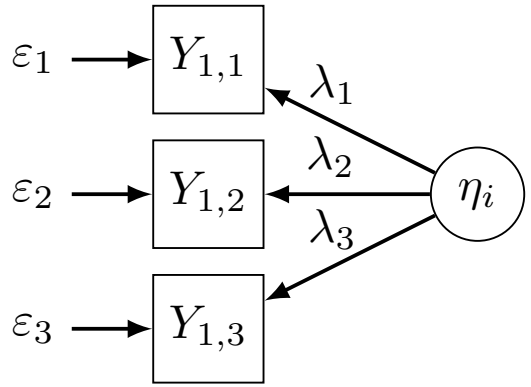
-----------------------------------------
```

Latent Variable Scaling

Marker-Variable Method



Effect-Coding Method



where

$$\frac{1}{K}\sum_{k=1}^{K}\lambda_k = 1$$

`semnova(..., ind_scaling = "first1", ...)`

- `ind_scaling`: Character String. Default is "`first1`" (marker-variable method). Can also be set to "`average1`" (effect-coding method).

Covariance Structures

Sphericity

$$\boldsymbol{\Sigma} = \begin{array}{l} \\ \text{intercept } (\pi_0) \\ \text{noise1 } (\pi_1) \\ \text{noise2 } (\pi_2) \\ \text{temp1 } (\pi_3) \\ \text{noise1:temp1 } (\pi_4) \\ \text{noise2:temp1 } (\pi_5) \end{array} \begin{array}{c} \begin{array}{cccccc} \text{intercept} & \text{noise1} & \text{noise2} & \text{temp1} & \text{noise1:temp1} & \text{noise2:tmp1} \end{array} \\ \begin{pmatrix} \sigma_{intercept} & & & & & \\ & \sigma_{noise} & 0 & & & \\ & 0 & \sigma_{noise} & & & \\ & & & \sigma_{temp} & & \\ & & & & \sigma_{noise:temp} & 0 \\ & & & & 0 & \sigma_{noise:temp} \end{pmatrix} \end{array}$$

```
semnova(..., sphericity = FALSE, ...)
```

- `sphericity`: Boolean. Default is `FALSE`. Imposes sphericity onto the model.

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Compound Symmetry

$$\boldsymbol{\Sigma} = \begin{array}{cc} & \begin{array}{cccccc} \text{none cold} & \text{low cold} & \text{high cold} & \text{none hot} & \text{low hot} & \text{high hot} \end{array} \\ \begin{array}{cc} \text{none} & \text{cold} \\ \text{low} & \text{cold} \\ \text{high} & \text{cold} \\ \text{none} & \text{hot} \\ \text{low} & \text{hot} \\ \text{high} & \text{hot} \end{array} & \begin{pmatrix} \sigma_a & \sigma_b & \sigma_b & \sigma_b & \sigma_b & \sigma_b \\ \sigma_b & \sigma_a & \sigma_b & \sigma_b & \sigma_b & \sigma_b \\ \sigma_b & \sigma_b & \sigma_a & \sigma_b & \sigma_b & \sigma_b \\ \sigma_b & \sigma_b & \sigma_b & \sigma_a & \sigma_b & \sigma_b \\ \sigma_b & \sigma_b & \sigma_b & \sigma_b & \sigma_a & \sigma_b \\ \sigma_b & \sigma_b & \sigma_b & \sigma_b & \sigma_b & \sigma_a \end{pmatrix} \end{array}$$

$$Var(Y_i) = Var(Y_j) \quad \forall i, j$$
$$Cov(Y_i, Y_j) = \rho \quad \forall i, j \text{ where } i \neq j$$

```
semnova(..., compound_symmetry = FALSE, ...)
```

- `compound_symmetry`: Boolean. Default is `FALSE`. Imposes compound symmetry onto the model.

Custom Contrasts / Hypotheses

$$\mathbf{C} = \begin{array}{ll} & \text{dependent variables} \\ \text{Factor} & \begin{array}{cccccc} \text{noise} & \text{none} & \text{low} & \text{high} & \text{none} & \text{low} & \text{high} \\ \text{temp.} & \text{cold} & \text{cold} & \text{cold} & \text{hot} & \text{hot} & \text{hot} \end{array} \\ \begin{array}{l} \text{contrast1} \\ \text{contrast1} \end{array} & \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{pmatrix} \end{array}$$

`lgc(...)`

- `C_matrix`: Constrast matrix. If not a square matrix, arbitrary orthogonal rows are added.
- `hypotheses`: List of integers vectors. Each element contains the row indices of the contrast matrix that are to be tested against zero.

Conclusion

Latent repeated measures ANOVA

- Extends the latent growth components approach
- Allows for latent variables in repeated measures analysis
- Allows for multi-factorial designs (of any size)
- Introduces SEM advantages to repeated measures ANOVA

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R package `semnova`

- Implements latent repeated measures ANOVA
- Performs various tests
  - Multivariate: Wald, Wilks, permutation test
  - Univariate: F-test, permutation test
- Imposes different error structures
- Uses different measurement models
- Tests custom contrasts / hypotheses

Conclusion

Next steps:

- Test for sphericity
- Test for measurement invariance
- Implementation of `anova()` function
- Interindividual differences (latent covariates)
- Implementation of between-subject designs within latent repeated measures ANOVA (e.g., EffectLiteR approach, Mayer et al., 2016)
- Small sample sizes (permutation test, Bayesian extension)
- Enhance output / be user-friendly
- Documentation / test cases

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Thank you for your attention!

`https://langenberg.github.io`

References

Mayer, A., Dietzfelbinger, L., Rosseel, Y., and Steyer, R. (2016). The EffectLiteR Approach for Analyzing Average and Conditional Effects. *Multivariate Behavioral Research*, 51(2-3):374–391.

Mayer, A., Steyer, R., and Mueller, H. (2012). A General Approach to Defining Latent Growth Components. *Structural Equation Modeling*, 19(4):513–533.

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Syndicus, M., Wiese, B. S., and van Treeck, C. (2016). In the Heat and Noise of the Moment: Effects on Risky Decision Making. *Environment and Behavior*, 50(1):3–27.