The apriori algorithm as an engine for computerized adaptive assessment

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PSYCHOCO, Dortmund
2020, February 28

Outline

Interest in alternative methods for adaptive testing

- Need for short self-report based assessments in health settings.

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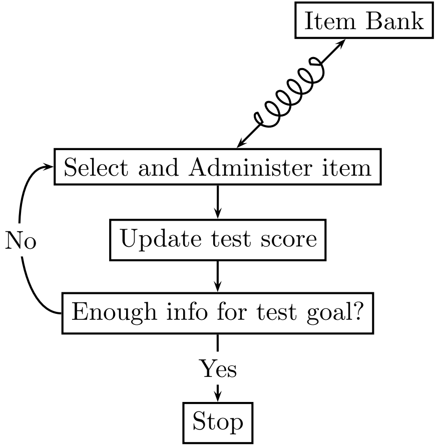
- Need for short self-report based assessments in health settings.
- Assessment often aimed at classification or prediction.
- Such tests require specific construction approaches (Smits et al., 2018; Oosterveld et al., 2019).
- Unfortunately, the standard approach under Item Response Theory is inappropriate.

Adaptive testing



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- Curtailment (a.k.a. ‘Countdown’, Butcher et al., 1985).
- Stochastic Curtailment (Finkelman et al., 2012, 2013; Fokkema et al., 2014; Smits & Finkelman, 2015).
- But:
  - Early stopping, i.e., no dynamic item selection.
  - Focus on (cumulative) sum scores.

Requirements for classification and prediction

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Would a rule learning algorithm like `apriori` be useful?

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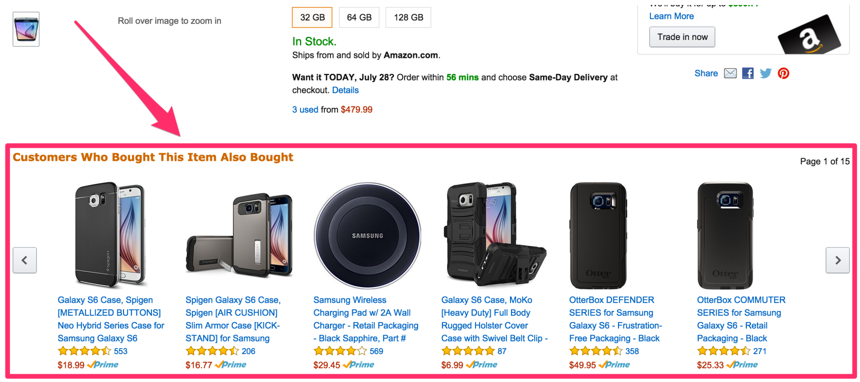

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- What items are frequently bought together?
- What symptoms frequently co-occur?

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$$L(\{\texttt{sleeping, eating}\} \Rightarrow \{\texttt{concentration}\}) = 4.00.$$

Building blocks

Requirements:

- Rule data base.
- Item selection.
- Test score.
- Stopping rule.

Rule data base

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- Unsupervised algorithm as supervised (Fürnkranz et al., 2012).
- Note that all variables are binary.

Item selection

- What item is most informative for criterion?
- Several statistics may be used:
  - Correlation (ϕ).
  - Odds-ratio.
  - Entropy.
  - .
- Each requires 2×2 table.

Required 2×2 table

Diagnosis

	$Y = 0$	$Y = 1$	
$X_j = 0, x_{i_1}, \ldots, x_{i_{k-1}}$	π_{00}	π_{01}	$1 - Q$
$X_j = 1, x_{i_1}, \ldots, x_{i_{k-1}}$	π_{10}	π_{11}	Q
	$1 - P$	P	

Cell probabilities obtainable from statistics

lhs		rhs	support	confidence	lift	count
{Back_R_Ankle}	=>	{crit0}	0.4478873	0.5530435	0.9647687	318
{Back_L_Knee}	=>	{crit0}	0.4309859	0.5303293	0.9251445	306
{Back_L_Wrist}	=>	{crit0}	0.4126761	0.5077990	0.8858409	293
{Back_R_Knee}	=>	{crit0}	0.4408451	0.5378007	0.9381781	313
{Back_L_Ankle}	=>	{crit0}	0.4492958	0.5452991	0.9512590	319
{Back_R_Wrist}	=>	{crit0}	0.4239437	0.5136519	0.8960512	301
{Back_L_Hip}	=>	{crit0}	0.4380282	0.5262267	0.9179877	311
{Back_R_Hip}	=>	{crit0}	0.4492958	0.5370370	0.9368459	319
{Front_L_Elbow}	=>	{crit0}	0.4605634	0.5351882	0.9336207	327
{Front_R_Elbow}	=>	{crit0}	0.4633803	0.5384615	0.9393309	329

Test score and stopping rule

Estimate of criterion probability after k items:

- $P(Y = 1 | x_{i_1}, \ldots, x_{i_k})$.
- $P(Y = 0) = 1 - P(Y = 1)$.

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Stopping rule:

- Set required certainty γ (e.g. 0.95).
- Stop if $P(Y = 1) > \gamma$ or if $P(Y = 1) < 1 - \gamma$.

Pseudo-code for training phase

1: Data.0 $\leftarrow$ Combine item set and criterion $=$ 0 into data base
Req.0 $\leftarrow$ Set requirements for rule quality in Data.0
Results.0 $\leftarrow$ Run `apriori` on Data.0 using Req.0
Rules.0 $\leftarrow$ Rules from Results.0 with criterion $=$ 0 as consequent
2: Data.1 $\leftarrow$ Combine item set and criterion $=$ 1 into data base
Req.1 $\leftarrow$ Set requirements for rule quality in Data.1
Results.1 $\leftarrow$ Run `apriori` on Data.1 using Req.1
Rules.1 $\leftarrow$ Rules from Results.1 with criterion $=$ 1 as consequent
3: Rules $\leftarrow$ Join Rules.0 and Rules.1

Pseudo-code for application phase

Require: Rules
Require: γ

PPV $\leftarrow$ 0
NPV $\leftarrow$ 0
Items.left $\leftarrow$ item set
Items.used $\leftarrow$ empty
while PPV$< \gamma$ **and** NPV$> 1 - \gamma$ **and** cardinality of items.left > 0 **do**
    Pattern $\leftarrow$ response pattern to Items.used
    Rules.s $\leftarrow$ rules with Pattern as sub pattern and cardinality + 1
    if Rules.s is not empty **then**
        Select item with highest statistic.
    else if Rules.s is empty **then**
        Select item randomly
    end if
    Administer item
    Remove item from Items.left
    Add item to Items.used
    PPV $\leftarrow P(Y = 1)$ given response pattern
    NPV $\leftarrow P(Y = 0)$ given response pattern
end while

Output: PPV, NPV, Items.used, Pattern.

Synthetic data

Prediction of criterion score using 17 symptoms

```
$prob.pos
            [,1]
 [1,] 0.07939914
 [2,] 0.08928571
 [3,] 0.08406114
 [4,] 0.12000000
 [5,] 0.14285714
 [6,] 0.13333333
 [7,]        NaN
 [8,]        NaN
 [9,]        NaN
[10,]        NaN
[11,]        NaN
[12,]        NaN
[13,]        NaN
[14,]        NaN
[15,]        NaN
[16,]        NaN
[17,]        NaN
```

```
$`in.basket`
  "n.MSA_Q_08" "MSA_Q_01"   "MSA_Q_02"
  "MSA_Q_15"   "MSA_Q_16"   "MSA_Q_06"
  "MSA_Q_03"   "MSA_Q_04"   "n.MSA_Q_05"
  "MSA_Q_07"   "MSA_Q_09"   "MSA_Q_10"
  "MSA_Q_11"   "n.MSA_Q_12" "n.MSA_Q_13"
  "MSA_Q_14"   "n.MSA_Q_17"
```

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- I have to re-evaluate.
- Do you have suggestions?

Thanks for your attention!

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