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Pavlo Blavatskyy

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Pavlo R. Blavatskyy

Institute of Public Finance
University of Innsbruck
Universitaetsstrasse 15
A-6020 Innsbruck
Austria

Phone: +43 (0) 512 507 71 56
Fax: +43 (0) 512 507 29 70
E-mail: Pavlo.Blavatskyy@uibk.ac.at

Abstract. Empirical research often requires a method how to convert a deterministic economic theory into an econometric model. A popular method is to add a random error term on the utility scale. This method, however, violates stochastic dominance. A modification of this method is proposed to avoid violations of dominance. The modified model compares favorably to other existing models in terms of goodness of fit to experimental data. The modified model can rationalize the preference reversal phenomenon. An intuitive axiomatic characterization of the modified model is provided. Important microeconomic concept of risk aversion is well-defined in the modified model.

Key Words: Decision Theory, Probabilistic Choice, Stochastic Dominance, Strong Utility, Risk Aversion, Preference Reversal Phenomenon

JEL Classification Codes: C25, D03, D81

An empirical researcher often encounters the following problem. On the one hand, economic theory assumes that a decision maker has a preference relation $\succsim$ over choice alternatives. In other words, implied choices are generically deterministic (except for a special case when a decision maker is indifferent). On the other hand, empirical data show that choices are often probabilistic.[1) In other words, revealed preferences are fuzzy (either imprecise, or random, or noisy). Thus, an empirical researcher wishing to relate economic theory to empirical data faces a major problem. How to extend a deterministic economic theory into a model of probabilistic choice?

One popular method is strong utility model that can be traced back to Fechner (1860). In economics, it was popularized by Hey and Orme (1994). Blavatskyy (2008) recently provided its axiomatic characterization. Several authors also extended the original model (*e.g.*, Hey, 1995; Buschena and Zilberman, 2000; Blavatskyy, 2007; Wilcox, 2008, 2010). Unfortunately, both the original model and its subsequent extensions violate first order stochastic dominance. Yet, such violations are rarely observed (*e.g.*, Carbone and Hey, 1995; Loomes and Sugden, 1998; Hey, 2001) and are normatively unappealing.

This paper presents a modification of strong utility (Fechner) model to avoid violations of stochastic dominance. First, the necessary notation is introduced in section 1. Strong utility (Fechner) model and its proposed modification are presented in section 2. Section 3 analyzes how the modified model fits experimental data. Section 4 takes a positive perspective on the modified model by demonstrating its ability to rationalize preference reversals (including strong reversals and reversals with probability equivalents). Section 5 takes a normative perspective on the modified model and provides its intuitive

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[1)] See Camerer (1989), Starmer and Sugden (1989), Hey and Orme (1994), Wu (1994), Ballinger and Wilcox (1997), Loomes and Sugden (1998), Hey (2001), Schmidt and Hey (2004), Schmidt and Neugebauer (2007).

axiomatic characterization. Section 6 shows that an important microeconomic concept of risk aversion is well-defined in the modified model (but not in the original strong utility model). Sections 3-6 are self-contained. The reader can skip any of these sections without the loss of continuity. Section 7 concludes.

1. Notation

Set X is a non-empty set of outcomes (consequences) that is totally ordered under a preference relation $\succsim$. Set X is not necessarily a subset of the Euclidian space $\mathbb{R}^n$. Lottery $L{:}X \to [0,1]$ is a probability distribution on set X, *i.e.*, $L(x)\in[0,1]$ for all $x\in X$ and $\Sigma_{x\in X} L(x)=1$. The set of all such lotteries is denoted by $\mathcal{L}$. A degenerate lottery that yields one outcome $x\in X$ with probability one is denoted by x. Notation $L\alpha L'$ denotes a probabilistic mixture that yields outcome $x\in X$ with probability $\alpha\cdot L(x)+(1-\alpha)\cdot L'(x)$, $\alpha\in[0,1]$.

For any lottery $L\in\mathcal{L}$, cumulative distribution function $F_L(x)$ is defined as

(1) $$F_L(x) = \Sigma_{y\in X,\, x\succsim y} L(y), \text{ for all } x\in X.$$

Similarly, for any $L\in\mathcal{L}$, decumulative distribution function $G_L(x)$ is defined as

(2) $$G_L(x) = \Sigma_{y\in X,\, y\succsim x} L(y), \text{ for all } x\in X.$$

For any L, $L'\in\mathcal{L}$, lottery $L\vee L'$ yields outcome $x\in X$ with a probability

(3) $$\min\{F_L(x),F_{L'}(x)\} + \max\{G_L(x),G_{L'}(x)\} - 1.$$

Lottery $L\vee L'$ is the least upper bound on lotteries L and L' in terms of first order stochastic dominance. Lottery $L\vee L'$ stochastically dominates both L and L' and there is no other lottery that stochastically dominates both L and L' but that is stochastically dominated by $L\vee L'$.

For any L, $L'\in\mathcal{L}$, lottery $L\wedge L'$ yields outcome $x\in X$ with a probability

(4) $$\max\{F_L(x),F_{L'}(x)\} + \min\{G_L(x),G_{L'}(x)\} - 1.$$

Lottery $L\wedge L'$ is the greatest lower bound on lotteries L and L' in terms of first order stochastic dominance. Both L and L' stochastically dominate lottery $L\wedge L'$ and there is no other lottery that is stochastically dominated by both L and L' but that stochastically dominates $L\wedge L'$.

2. Strong Utility (Fechner) Model and Its Modification

We assume that a decision maker has a rational continuous preference relation $\succsim$ on $\mathscr{L}$. Such preference admits utility representation:

(5) $$L \succsim L' \text{ if and only if } U(L) - U(L') \geq 0,$$

where $U : \mathscr{L} \to \mathbb{R}$ is a real-valued utility function.

Strong utility (Fechner) model postulates that a decision maker chooses lottery L over lottery L' if

(6) $$U(L) - U(L') \geq \xi,$$

where ξ is a random variable (with zero mean) that is independently and identically distributed across all lottery pairs. In the original strong utility model, error term ξ is homoscedastic (*e.g.*, Hey and Orme, 1994). In the subsequent extensions, error term ξ is heteroscedastic (*e.g.*, Hey, 1995; Buschena and Zilberman, 2000).

Model (6) violates stochastic dominance. Indeed, it even violates strict dominance. Yet, a simple modification of model (6) cures the problem.

First, consider the case when lottery L stochastically dominates lottery L'. In this case, $U(L) - U(L') = U(L \vee L') - U(L \wedge L')$. Thus, to avoid violations of stochastic dominance, we need to make sure that the realization of a random variable ξ is never greater than the difference $U(L \vee L') - U(L \wedge L')$. In other words, inequality (6) must be always satisfied if L stochastically dominates L'. Thus, stochastic dominance imposes an upper bound on possible errors:

(7) $$\xi \leq U(L \vee L') - U(L \wedge L').$$

Second, consider the case when L' stochastically dominates L. In this case, $U(L) - U(L') = U(L \wedge L') - U(L \vee L')$. To avoid violations of stochastic dominance, we need to make sure that the realization of a random variable ξ is never less than the difference $U(L \wedge L') - U(L \vee L')$. In other words, inequality (6) must always hold with a reversed sign if L' stochastically dominates L. Thus, stochastic dominance also imposes a lower bound on possible errors:

(8) $$\xi \geq U(L \wedge L') - U(L \vee L').$$

Inequalities (7) and (8) imply that random variable ξ must be distributed on a bounded interval. In general, this interval varies across lottery pairs. Thus, random variable ξ cannot be independently and identically distributed across all lottery pairs. Yet, it is possible to write random error ξ as $\epsilon\cdot[U(L\vee L')-U(L\wedge L')]$. Inequalities (7) and (8) are then both satisfied if random variable ϵ is independently and identically distributed on the interval [-1,1].

Using error term $\xi=\epsilon\cdot[U(L\vee L')-U(L\wedge L')]$ not only imposes stochastic dominance but also resolves another problem. In the original Fechner model (6), error term ξ is added on the *absolute* utility scale. In most decision theories, utility function $U(.)$ is unique only up to a positive affine transformation. Multiplying utility function $U(.)$ by an arbitrary positive constant makes no difference for representation (5) but it does affect the distribution of random errors in model (6). In contrast, random error ϵ is added on the *relative* utility scale. Thus, multiplying utility function $U(.)$ by an arbitrary positive constant does not affect the distribution of random error ϵ.

To summarize, strong utility (Fechner) model (6) is modified so that a decision maker chooses lottery L over lottery L' if

$$U(L) - U(L')\geq\epsilon\cdot[U(L\vee L')-U(L\wedge L')], \tag{9}$$

where ϵ is a random variable symmetrically distributed around zero on the interval [-1,1]. We need to distinguish two cases. First, if $U(L\vee L')\neq U(L\wedge L')$, then both sides of inequality (9) can be divided by $U(L\vee L')-U(L\wedge L')$. Second, it is possible that $U(L\vee L')=U(L\wedge L')$.[2] In this case, $U(L)=U(L')$ and inequality (9) holds trivially as equality. In other words, a decision maker can choose both L and L'. For simplicity, we assume that both lotteries are chosen with probability 0.5 in this case.[3]

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[2] If a decision maker has a strict preference over all outcomes (for any $x, y\in X$ either $x\succ y$ or $y\succ x$), then the second case is possible only when $L=L'$.

[3] Alternatively, choice probability may be left undefined in this case.

Let F: $[-1,1]\rightarrow[0,1]$ be the cumulative distribution function of random error ϵ. Function $F(.)$ can be any non-decreasing function satisfying the restriction $F(v)+F(-v)=1$ for all $v\in[-1,1]$. A decision maker then chooses lottery L over lottery L' with probability (10).

$$(10)\qquad \begin{aligned} &P(L,L')=F\left(\frac{U(L)-U(L')}{U(L\vee L')-U(L\wedge L')}\right), && \text{if } U(L\vee L')\neq U(L\wedge L') \\ &P(L,L')=0.5, && \text{if } U(L\vee L')=U(L\wedge L') \end{aligned}$$

According to formula (10), $P(L,L')=F(1)$ if L stochastically dominates L' (in this case $L\vee L'=L$ and $L\wedge L'=L'$) and $P(L,L')=F(-1)=1-F(1)$ if L is stochastically dominated by L'. Thus, by setting $F(1)=1$[4] we avoid violations of stochastic dominance.

Formula (10) also implies that $P(L,L')\geq 0.5$ if $U(L)\geq U(L')$. Thus, model (10) satisfies weak stochastic transitivity: if $P(L,L')\geq 0.5$ and $P(L',L'')\geq 0.5$ then $P(L,L'')\geq 0.5$. This gives model (10) a comparative advantage over random preference approach (*e.g.*, Falmagne, 1985; Loomes and Sugden, 1995) including random utility (*e.g.*, Gul and Pesendorfer, 2006). Random preference/utility approach allows for intransitive choice cycles (similar to the Condorcet's paradox) that are normatively unappealing and rarely observed in the data (*e.g.*, Rieskamp et al., 2006, p. 648).

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[4] Setting $F(1)=1$ is appealing on normative grounds. From a descriptive perspective, it may be desirable to set $F(1)<1$. This allows model (10) to account for rare violations of stochastic dominance that are observed in the data. See Loomes et al. (2002) for a more detailed discussion.

3. Fit to Experimental Data

Decision theories are usually evaluated according to their goodness of fit to experimental data (*e.g.*, Hey and Orme, 1994; Harless and Camerer, 1994). However, such practice critically depends on the method of converting a deterministic theory into a model of stochastic choice (see Hey (2005), Loomes (2005), Blavatskyy and Pogrebna (2010)). In this section we show that model (10) improves upon model (6) in terms of goodness of fit to experimental data. We also compare model (10) to other existing models.

We use experimental data collected by Hey (2001), which is the largest data set of its kind. In this data set, 53 individuals faced 100 binary choice problems. Each choice problem was repeated five times. Each choice problem involved lotteries over four outcomes: -£25, £25, £75 and £125.

First, we consider the case when function $U(.)$ is von Neumann-Morgenstern expected utility function, *i.e.* $U(L) = \Sigma_{x\in X} L(x)\cdot u(x)$, where $u: X\rightarrow\mathbb{R}$ is (Bernoulli) utility function. Bernoulli utility function can be normalized for any two outcomes. We use normalization $u(-£25)=0$ and $u(£125)=1$. Utilities of two other outcomes $u(£25)$ and $u(£75)$ remain subjective parameters to be estimated.

We assume that function $F(.)$ in model (10) is the cumulative distribution function of the normal distribution with zero mean and constant variance $\sigma>0$. Estimation is done separately for each individual by maximizing total log-likelihood (equation (10) shows the likelihood of one decision). For each subject we estimate three parameters: $u(£25)$, $u(£75)$ and σ. Non-linear optimization is solved in the Matlab 7.2 package (based on the Nelder-Mead simplex algorithm). Program code and data are available from the author on request.

To evaluate the relative goodness of fit, we compare model (10) to four other models of probabilistic choice. First, we estimate the original strong utility (Fechner) model (6). We assume that random error ξ in model (6) is normally distributed with zero mean and constant variance. Thus, a decision maker chooses lottery L over lottery L' with probability $P(L,L')=F(U(L) - U(L'))$, where function $F(.)$ is the cumulative distribution function of the normal distribution with zero mean and constant variance $\sigma>0$.

Second, we estimate a contextual utility model of Wilcox (2008, 2010). In this model, a decision maker chooses lottery L over lottery L' with probability $P(L,L')=F([U(L) - U(L')]/[u(z) - u(y)])$, where function $F(.)$ is the cumulative distribution function of the normal distribution with zero mean and constant variance $\sigma>0$ and outcomes $z\in X$ and $y\in X$ are correspondingly the best and the worst possible outcome in lotteries L and L'.

Third, we estimate incremental expected utility advantage model using formula (2) in Fishburn (1978, p.635) with function $\rho(v)=v^{\mu}$ for all $v\geq 0$ and μ being a subjective parameter. Fourth, we estimate the model of Blavatskyy (2009, 2011) using formula (3) in Blavatskyy (2011) with function $\phi(x)=e^{\lambda v}-1$ for all $v\geq 0$ and λ being a subjective parameter.

The models of Fishburn (1978) and Blavatskyy (2009, 2011) rule out violations of stochastic dominance. Yet, Hey (2001) found 24 violations of stochastic dominance in 1590 choice decisions (rate of violation 1.5%). Thus, the models of Fishburn (1978) and Blavatskyy (2009, 2011) can be estimated on Hey (2001) data set only if we introduce the possibility of trembles (see Loomes et al. (2002) for a more detailed discussion). Specifically, a decision maker chooses lottery L over lottery L' with probability

(11) $$(1-\tau)\cdot P(L,L') + \tau\cdot[1-P(L,L')],$$

where $P(L,L')$ denotes the probability that L is chosen over L' in a model of probabilistic choice without trembles, and $\tau\in[0,0.5]$ is a subjective parameter interpreted as the probability of a tremble (or lapse of concentration).

For each subject, five models are compared in terms of their goodness of fit to the subject's revealed choice pattern. We use Vuong likelihood ratio test for strictly non-nested models (see Vuong (1989) and Appendix A.2 in Loomes et al. (2002) for technical details). The models of Fishburn (1978) and Blavatskyy (2009, 2011) have one extra parameter compared to models (6), (10) and a contextual utility model of Wilcox (2008, 2010). To penalize the models of Fishburn (1978) and Blavatskyy (2009, 2011) for one extra parameter we use two standard correction factors: Akaike and Schwarz information criteria (see Vuong (1989) p. 318 for technical details). Tables 1 and 2 summarize the results correspondingly for Akaike and Schwarz information criteria.

Model (6)	Wilcox (2008)	Fishburn (1978)	Blavatskyy (2011)
22 (42%)/0 (0%)	10 (19%)/0 (0%)	7 (13%)/1 (2%)	1 (2%)/6 (11%)

Table 1 Number and percentage of subjects for whom model (10) fits significantly better (nominator) and significantly worse (denominator) than the corresponding model in the first row when *U*(.) is expected utility function and Akaike information criterion is used (at 1% significance level).

Model (6)	Wilcox (2008)	Fishburn (1978)	Blavatskyy (2011)
22 (42%)/0 (0%)	10 (19%)/0 (0%)	19 (36%)/1 (2%)	2 (4%)/4 (8%)

Table 2 Number and percentage of subjects for whom model (10) fits significantly better (nominator) and significantly worse (denominator) than the corresponding model in the first row when *U*(.) is expected utility function and Schwarz information criterion is used (at 1% significance level).

Tables 1-2 show that modified strong utility (Fechner) model (10) clearly improves upon the original model (6), contextual utility model of Wilcox (2008, 2010) and the model of Fishburn (1978). Yet, model (10) appears to be inferior to the model of Blavatskyy (2009, 2011). The latter can account for certain types of the common ratio effect (*e.g.*, Loomes, 2005) and violations of betweenness (*e.g.*, Blavatskyy, 2006) even if function *U*(.) is expected utility function. In contrast, model (10) can accommodate such violations of expected utility theory only if function *U*(.) is non-linear in probabilities. Since people often violate expected utility, it is not surprising that the model of Blavatskyy (2009, 2011) fits the data better than does model (10).

Since model (10) is handicapped compared to the model of Blavatskyy (2009, 2011) when $U(.)$ is expected utility function, it is interesting to repeat the same estimation as described above with $U(.)$ being rank-dependent utility, *i.e.* $U(L) = \Sigma_{x\in X}\, u(x)\cdot[w(G_L(x))-w(1-F_L(x))]$, where $u: X\to\mathbb{R}$ is (Bernoulli) utility function and w: $[0,1]\to[0,1]$ is probability weighting function. We use Quiggin (1981) probability weighting function $w(p)=p^\gamma/[p^\gamma+(1-p)^\gamma]^{1/\gamma}$, where $\gamma>0$ is a subjective parameter. Expected utility is a special case of rank-dependent utility when function $w(.)$ is linear ($\gamma=1$). Tables 3 and 4 summarize the results correspondingly for Akaike and Schwarz information criteria.

Model (6)	Wilcox (2008)	Fishburn (1978)	Blavatskyy (2011)
26 (49%)/0 (0%)	13 (25%)/0 (0%)	6 (11%)/1 (2%)	3 (6%)/5 (9%)

Table 3 Number and percentage of subjects for whom model (10) fits significantly better (nominator) and significantly worse (denominator) than the corresponding model in the first row when $U(.)$ is rank-dependent utility function and Akaike information criterion is used (at 1% significance level).

Model (6)	Wilcox (2008)	Fishburn (1978)	Blavatskyy (2011)
26 (49%)/0 (0%)	13 (25%)/0 (0%)	16 (30%)/1 (2%)	3 (6%)/3 (6%)

Table 4 Number and percentage of subjects for whom model (10) fits significantly better (nominator) and significantly worse (denominator) than the corresponding model in the first row when $U(.)$ is rank-dependent utility function and Schwarz information criterion is used (at 1% significance level).

Tables 3-4 confirm our previous conclusion for expected utility: model (10) improves upon the original model (6), contextual utility model of Wilcox (2008, 2010) as well as the model of Fishburn (1978). Tables 3-4 also confirm our intuition about the handicap of model (10) when combined with expected utility function. For rank-dependent utility, unlike for expected utility, model (10) achieves about a similar goodness of fit as the model of Blavatskyy (2011).

Model (10) fits well to experimental data due to a simple reason. When error term ϵ is normally distributed, model (10) produces a relatively high choice variability when none of the two lotteries dominates the other. At the same time, it also allows for rare violations of stochastic dominance when one of the lotteries dominates the other (see footnote 4). And this is exactly how experimental data look like (*e.g.*, Loomes and Sugden (1998), Hey (2001)). To the best of my knowledge, model (10) is the first model of probabilistic choice that mimics this feature of empirical data.

In contrast, the original strong utility (Fechner) model (6) is too simplistic. This model produces a relatively high choice variability irrespective of the fact whether one of the two lotteries dominates the other or not. Effectively, this model "overpredicts" probabilistic choice (see also discussion in Loomes and Sugden (1998)).

On the other hand, the models of Fishburn (1978) and Blavatskyy (2009, 2011) are too restrictive. These models completely rule out violations of stochastic dominance. To account for rare violations of stochastic dominance in the data, these two models must be artificially "augmented" with an extra tremble parameter (*cf.* equation (11)). We should note in passing that the same criticism also applies to a random preference approach including random expected utility (see also discussion in Loomes and Sugden (1998)).

Contextual utility model of Wilcox (2008, 2010) goes only half-way. This model produces a relatively high choice variability when at least one of the two lotteries is non-degenerate. At the same time, it also allows for rare violations of (strict) dominance when both lotteries are degenerate. Thus, this model makes a step in the right direction by moving away from the complete insensitivity of the original strong utility (Fechner) model (6). Yet, contextual utility model of Wilcox (2008, 2010) does not go far enough. In particular, it does not single out instances when one of the two lotteries stochastically dominates the other.

4. The Preference Reversal Phenomenon

The preference reversal phenomenon is one of the most robust behavioral regularities (see Seidl (2002) for a recent review). Deterministic decision theories based on a rational preference relation cannot account for this phenomenon. Original strong utility (Fechner) model (6) combined with a plausible definition of a probabilistic certainty equivalent (see definition 1 below) also rules out systematic preference reversals. In contrast, as shown in this section, model (10) can rationalize the preference reversal phenomenon.

The preference reversal phenomenon is demonstrated with two lotteries of a similar expected value. Lottery L (called the P-bet) yields a modest payoff of y dollars with a probability $L(y)$ close (but not equal) to one and zero dollars otherwise. Lottery L' (called the \$-bet) yields a relatively large payoff of z dollars, $z>y$, with a small probability $L'(z)$, $L'(z)<L(y)$, and zero dollars otherwise. A standard preference reversal is observed when a decision maker chooses L over L' but reveals a higher certainty equivalent for L' than for L. A nonstandard preference reversal occurs when a decision maker chooses L' over L but reveals a higher certainty equivalent for L than for L'.

According to any deterministic decision theory based on a rational preference relation, both types of preference reversals should never occur (except in a special case when $L \sim L'$). However, empirical evidence shows that standard preference reversals usually significantly outnumber nonstandard preference reversals (*e.g.*, Tversky et al., 1990). This robust behavioral regularity became known as the preference reversal phenomenon.

To analyze the phenomenon within a model of probabilistic choice, it is necessary to define a probabilistic certainty equivalent. Blavatskyy (2009) proposed the following definition. The probability that the certainty equivalent of a lottery is less than or equal to some amount x is simply the probability that the amount x is chosen over the lottery in a direct binary choice.

Definition 1. A probabilistic certainty equivalent of lottery L is a random variable with a cumulative distribution function $P(x,L)$.

Given Definition 1, the likelihood of observing a standard preference reversal is given by $[1-P(L',L)]\cdot\int P(x,L)\mathrm{d}P(x,L')$ and the likelihood of observing a nonstandard preference reversal is given by $P(L',L)\cdot[1-\int P(x,L')\mathrm{d}P(x,L)]$. Hence, standard preference reversals occur more frequently if

$$\int P(x,L)\mathrm{d}P(x,L')>P(L',L). \tag{12}$$

Let us first consider original strong utility (Fechner) model (6). If lotteries L and L' yield the same utility (*i.e.*, $U(L)=U(L')$), then the left hand side and the right hand side of inequality (12) are both equal to 0.5. Hence, in this case, systematic preference reversals cannot happen.

Let us now consider modified strong utility (Fechner) model (10). According to formula (10), $P(x,L)=F(-1)$ for any $x\leq 0$ and $P(x,L)=F(1)$ for any $x\geq y$. Similarly, $P(x,L')=F(-1)$ for any $x\leq 0$ and $P(x,L')=F(1)$ for any $x\geq z$. Using these results, inequality (12) can be rewritten as the following condition:

$$\int_0^y P(x,L)\mathrm{d}P(x,L')>P(L',L)+F(1)\cdot P(y,L')-F(1)^2. \tag{13}$$

If binary choice probabilities are degenerate, *i.e.* if there is an amount $CE(L)$ such that $P(x,L)=0$ for all $x<CE(L)$ and $P(x,L)=1$ for all $x>CE(L)$, then both sides of condition (13) are equal and it cannot hold with strict inequality. In other words, a deterministic decision theory based on a rational preference relation cannot account for the preference reversal phenomenon. Yet, if choice probabilities are non-degenerate, condition (13) may be fulfilled.

For example, consider model (10) when $U(.)$ is expected utility function. For simplicity, we normalize Bernoulli utility function so that $u(0)=0$ and $u(z)=1$. As before, we consider the case when lotteries L and L' yield the same expected utility (*i.e.*, $L(y)\cdot u(y)=L'(z)$ and $P(L',L)=0.5$).

Lottery $x\vee L$ yields outcome x with probability $1-L(y)$ and outcome y with probability $L(y)$. Hence, expected utility of lottery $x\vee L$ is equal to

(14) $$U(x \vee L) = [1 - L(y)] \cdot u(x) + L(y) \cdot u(y).$$

Lottery $x \wedge L$ yields outcome x with probability $L(y)$ and nothing otherwise. Hence, expected utility of lottery $x \wedge L$ is equal to

(15) $$U(x \wedge L) = L(y) \cdot u(x).$$

According to formula (10), probability $P(x,L)$ is given by

(16) $$P(x,L) = F\left(\frac{u(x) - L(y) \cdot u(y)}{[1 - 2L(y)] \cdot u(x) + L(y) \cdot u(y)}\right), \qquad \text{for all } x < y.$$

Similarly, we can show that binary choice probability $P(x,L')$ is given by

(17) $$P(x,L') = F\left(\frac{u(x) - L'(z)}{[1 - 2L'(z)] \cdot u(x) + L'(z)}\right), \qquad \text{for all } x < y.$$

Consider the simplest possible case when random error ϵ is uniformly distributed on the interval [-1,1]. In this special case, model (10) can be viewed as a formalization of intuitive ideas of MacCrimmon and Smith (1986) that were recently reiterated in Butler and Loomes (2007). If random error ϵ is uniformly distributed, then its cumulative distribution function $F(.)$ is a linear function, *i.e.* $F(v) = 0.5 + 0.5v$. Using this result, we can plug equations (16) and (17) into condition (13) to obtain a simplified condition:

(18) $$\ln\left(\frac{1 + L(y) - 2L'(z)}{1 - L(y)}\right) > \frac{2 \cdot [1 - L(y)] \cdot [L'(z) - L(y)]}{[1 - L'(z)] \cdot [1 + L(y) - 2L'(z)]}.$$

The left (right) hand side of inequality (18) is always positive (negative) if $L(y) > L'(z)$. Thus, condition (18) is always satisfied. In other words, model (10) can generate systematic preference reversals even though model (6) cannot.

Why do systematic preference reversals emerge in modified strong utility (Fechner) model (10) but not in the original model (6)? The intuition is simple. If lotteries L and L' yield the same expected utility, then both the original and modified strong utility (Fechner) model postulate that $P(L',L) = 0.5$. Definition 1 then implies that median certainty equivalents of lotteries L and L' are the same. This implication again holds for both models.

The two models diverge in their assumptions about the distribution of certainty equivalents of L and L'. In original model (6), certainty equivalents of L and L' are symmetrically distributed. In contrast, in modified model (10), certainty equivalents of L and L' are skewed due to the bounds imposed by stochastic dominance. Specifically, the certainty equivalent of L is distributed on the interval $[0,y]$ and the certainty equivalent of L' — on the interval $[0,z]$.[5] Thus, the certainty equivalent of L is negatively skewed. At the same time, the certainty equivalent of L' is positively skewed. In other words, the certainty equivalent of L' is more likely to be the greater of the two. Hence, standard preference reversals are more likely to be observed than nonstandard ones.

Model (10) can rationalize not only a higher incidence of standard preference reversals but also the existence of strong reversals (Fishburn, 1988, p.46). Strong reversals occur when an individual chooses lottery L over lottery L' in a direct binary choice but reveals a certainty equivalent for L' which is greater than the highest possible outcome in lottery L (*i.e.*, y). According to Definition 1, the likelihood of the certainty equivalent of L' to be greater than y is simply $P(L',y)$. Hence, strong reversals occur with probability $P(L',L)\cdot P(L',y)$.

Note that if lotteries L and L' yield the same expected utility, then the likelihood of observing strong reversals cannot be greater than 0.25 because $P(L',L)=0.5$ and $P(L',y)<P(L',L)$. Experimental data appear to support this prediction (*e.g.*, Butler and Loomes,2007).

Butler and Loomes (2007) recently found evidence of nonstandard preference reversals with probability equivalents. Model (10) can account for such reversals if we use a plausible definition of probabilistic probability equivalents (see definition 2 in Blavatskyy (2009)).

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[5] See also graphical representation in MacCrimmon and Smith (1986) and Butler and Loomes (2007).

5. Axiomatic Characterization

This section provides a simple axiomatization of model (10) when $U(.)$ is von Neumann–Morgenstern expected utility function. The primitive of choice is a binary choice probability function $P:\mathcal{L}\times\mathcal{L}\rightarrow[0,1]$. Probability $P(L,L')$ is observable from a relative frequency of L choices when an individual chooses repeatedly between L and L', $L\neq L'$. Probability $P(L,L)$ is not observable from a direct binary choice. For simplicity, we assume that $P(L,L)=0.5$ for all $L\in\mathcal{L}$.

First, we impose standard axioms on function $P:\mathcal{L}\times\mathcal{L}\rightarrow[0,1]$. Axioms 1–4 are probabilistic analogs of the corresponding axioms in expected utility theory.

Axiom 1 (*Probabilistic Completeness*) $P(L,L')+P(L',L)=1$ for all $L,L'\in\mathcal{L}$.

Axiom 2 (*Weak Stochastic Transitivity*) If $P(L,L')\geq0.5$ and $P(L',L'')\geq0.5$ then $P(L,L'')\geq0.5$ for all L, L', $L''\in\mathcal{L}$.

Axiom 3 (*Probabilistic Continuity*) The sets $\{\alpha\in[0,1]: P(L\alpha L',L'')\geq0.5\}$ and $\{\alpha\in[0,1]: P(L'',L\alpha L')\geq0.5\}$ are closed for all L, L', $L''\in\mathcal{L}$.

Axiom 4 (*Probabilistic Independence Axiom*) $P(L,L')=P(L\alpha L'', L'\alpha L'')$ for all L, L', $L''\in\mathcal{L}$ and $\alpha\in(0,1]$.

Proposition 1 (*von Neumann and Morgenstern, 1944*) If Axioms 1–4 hold then there is a utility function $u:X\rightarrow\mathbb{R}$ such that for any $L,L'\in\mathcal{L}$:

(19) $$P(L,L')\geq0.5 \quad \text{if and only if} \quad \Sigma_{x\in X}L(x)\cdot u(x)\geq\Sigma_{x\in X}L'(x)\cdot u(x).$$

Proof Define an auxiliary preference relation $L\succsim_{0.5}L'$ if $P(L,L')\geq0.5$. If Axioms 1–4 hold then preference relation $\succsim_{0.5}$ satisfies all the axioms of expected utility theory. Proposition 1 then immediately follows from expected utility theorem of von Neumann and Morgenstern (1944) *Q.E.D.*

Consider two arbitrary lotteries L and L'. According to axiom 3, the sets $\{\alpha\in[0,1]: P([L\vee L']\alpha[L\wedge L'],L)\geq0.5\}$ and $\{\alpha\in[0,1]: P(L,([L\vee L']\alpha[L\wedge L'])\geq0.5\}$ are both closed. Both of these sets are non-empty ($\alpha=1$ always belongs to the first set and $\alpha=0$ always belongs to the second set). Since set [0,1] is connected, there exists at least one probability $\alpha_{L,L'} \in [0,1]$ that belongs to both of these sets, *i.e.*, $P(L, [L \vee L']\alpha_{L,L'}[L \wedge L'])=0.5$.

Two cases are possible. First, if the expected utility of lottery $L \vee L'$ is strictly greater than the expected utility of lottery $L \wedge L'$, then probability $\alpha_{L,L'}$ is unique. In particular, proposition 1 implies that probability $\alpha_{L,L'}$ is given by (20).

$$\alpha_{L,L'} = \frac{\sum_{x \in X} L(x) \cdot u(x) - \sum_{x \in X} [L \wedge L'](x) \cdot u(x)}{\sum_{x \in X} [L \vee L'](x) \cdot u(x) - \sum_{x \in X} [L \wedge L'](x) \cdot u(x)} \tag{20}$$

Second, if expected utility of lottery $L \vee L'$ is equal to the expected utility of lottery $L \wedge L'$, then probability $\alpha_{L,L'}$ is not unique. In fact, in this case, any probability $\alpha_{L,L'} \in [0,1]$ satisfies condition $P(L, [L \vee L']\alpha_{L,L'}[L \wedge L'])=0.5$. For simplicity, we assume that $\alpha_{L,L'}=0.5$ in this case.

Intuitively, we can think of probability $\alpha_{L,L'}$ as a contextual probability equivalent of lottery L given that an individual faces a binary choice between lotteries L and L'. If $\alpha_{L,L'}=1$ then lottery L stochastically dominates lottery L'. If $\alpha_{L,L'}=0$ then lottery L is stochastically dominated by lottery L'. If $\alpha_{L,L'}=0.5$ then lotteries L and L' yield the same expected utility and $P(L,L')=0.5$. More generally, the higher is probability $\alpha_{L,L'}$ the more an individual is likely to choose lottery L over lottery L'.

Contextual probability equivalent of lottery L' is defined in a similar vein. It is the probability $\alpha_{L',L} \in [0,1]$ such that $P(L', [L \vee L']\alpha_{L',L}[L \wedge L'])=0.5$. Note that $\alpha_{L',L}=1-\alpha_{L,L'}$. Our last axiom postulates that the likelihood of choosing lottery L over lottery L' depends only on the difference in contextual probability equivalents $\alpha_{L,L'}-\alpha_{L',L}$.

Axiom 5 (*Probabilistic Choice*) $P(L,L')=P(L'', L''')$ for all $L, L', L'', L''' \in \mathcal{L}$ such that $\alpha_{L,L'}-\alpha_{L',L}=\alpha_{L'',L'''}-\alpha_{L''',L''}$.

Axiom 5 follows as a corollary from Proposition 1 in a special case when function $P(.,.)$ takes only three values: 0, 0.5 or 1.[6)] When choices are truly probabilistic, it does not follow from axioms 1-4 and we must add it to the list.

Proposition 2 Binary choice probability function $P:\mathcal{L}\times\mathcal{L}\rightarrow[0,1]$ satisfies axioms 1-5 if and only if there is a utility function $u:X\rightarrow\mathbb{R}$ such that for all $L,L'\in\mathcal{L}$ choice probability $P(L,L')$ is given by formula (10) with $U(.)$ being von Neumann-Morgenstern expected utility function, *i.e.* $U(L) = \Sigma_{x\in X} L(x)\cdot u(x)$, and F: $[-1,1]\rightarrow[0,1]$ being an arbitrary non-decreasing function satisfying the restriction $F(v)+F(-v)=1$ for all $v\in[-1,1]$.

Proof The necessity of axioms 1-5 follows by simple algebra from formula (10). To prove sufficiency, note that axiom 5 effectively postulates that $P(L,L')=F(\alpha_{L,L'}-\alpha_{L',L})$, where F: $[-1,1]\rightarrow[0,1]$ is an arbitrary function. Plugging the definition of contextual probability equivalents (20) into the last expression immediately yields formula (10). *Q.E.D.*

Axiom 5 assumes that binary choice probabilities depend on the *difference* in contextual probability equivalents. Alternatively, one can assume that they depend on the *ratio* of contextual probability equivalents. It turns out that this assumption together with axioms 1-4 provide a new axiomatization of incremental expected utility advantage model (Fishburn, 1978). Thus, model (10) is also related to the model of Fishburn (1978). These two models diverge only in the arithmetic assumption about what drives binary choice probabilities: the difference or the ratio of contextual probability equivalents. Econometric estimation in section 3 suggests that the difference assumption (*i.e.*, axiom 5) provides a better fit to experimental data than does the ratio assumption.

[6)] This special case may be considered as nearly-conventional economic theory based on a revealed preference relation: $L'\succ L$ when $P(L,L')=0$, $L\succ L'$ when $P(L,L')=1$, and $L\sim L'$ when $P(L,L')=0.5$. This is "nearly-conventional" because in standard economic theory a choice probability $P(L,L')$ is not defined when $L\sim L'$.

6. Risk Aversion

In this section, we consider two people: an individual ♀ characterized by a binary choice probability function $P_{♀}:\mathcal{L}\times\mathcal{L}\rightarrow[0,1]$ and an individual ♂ characterized by a function $P_{♂}:\mathcal{L}\times\mathcal{L}\rightarrow[0,1]$. The notion of relative risk aversion can be introduced by direct analogy to Yaari's (1969) acceptance sets. A more risk averse individual is always at least as likely to choose a degenerate lottery over a risky lottery as is a less risk averse individual.

Definition 2 An individual ♀ is probabilistically *more risk averse than* an individual ♂ if $P_{♀}(x,L)\geq P_{♂}(x,L)$ for all $x\in X$ and all $L\in\mathcal{L}$ with a strict inequality for at least one outcome $x\in X$ and one lottery $L\in\mathcal{L}$.

Definition 2 of the more-risk-averse-than relation between individuals is quite general. It does not require outcomes to be measurable in real numbers. It also imposes no restrictions on a binary choice probability function. Thus, definition 2 is applicable to any model of probabilistic choice. In particular, we can investigate implications of Definition 2 for model (6) and model (10).

Definition 2, however, has one immediate implication for all models of probabilistic choice. We can unambiguously rank two individuals in terms of their risk attitudes only if they choose in identical manner between riskless alternatives (degenerate lotteries). If this is not the case, heterogeneous risk attitudes are confounded with heterogonous tastes over outcomes and we cannot make a clear comparison of individuals in terms or relative risk aversion.

For example, suppose that individual ♀ prefers apples to oranges and individual ♂ has the opposite preference. Furthermore, suppose that individual ♀ chooses one apple for sure over a 50% chance of two oranges (nothing otherwise) and individual ♂ makes the opposite choice. We cannot conclude that individual ♀ is more risk averse than individual ♂. Individual ♀ could have chosen an apple because she likes apples, not because she is averse to risk.

Since Definition 2 implies that individuals ♀ and ♂ have the same preference ordering over outcomes, we can write that set X is totally ordered under a preference relation $\succsim$ without any individual specific subscripts. As in the previous two sections, we consider models (6) and (10) when $U(.)$ is von Neumann–Morgenstern expected utility function. First, we state impossibility result for model (6).

Blavatskyy (2011a) shows that it is impossible to rank individuals in terms of probabilistic risk aversion under original strong utility (Fechner) model (6). Wilcox (2010) discusses the failure of strong utility (Fechner) model to capture risk aversion in the context of lotteries over monetary outcomes. This failure is not surprising if we look at the axiomatic characterization of strong utility (Blavatskyy, 2008). One of its axioms (interchangeability) postulates that risky lotteries can be replaced by their certainty equivalents without affecting choice probabilities. Yet, such exchange operation inevitably leads to the loss of information about risk preferences.

Proposition 3 Under modified strong utility (Fechner) model (10) an individual ♀ is probabilistically more risk averse than an individual ♂ if $F_{♀}(v)=F_{♂}(v)$ for all $v\in[-1,1]$, inequality (21) holds for all x, y, $z\in X$ such that $z\succsim x\succsim y$ and it holds as strict inequality for at least one triple of outcomes x, y, $z\in X$, $z\succsim x\succsim y$.

$$\frac{u_{♀}(x) - u_{♀}(y)}{u_{♀}(z) - u_{♀}(x)} \geq \frac{u_{♂}(x) - u_{♂}(y)}{u_{♂}(z) - u_{♂}(x)} \tag{21}$$

Proof is presented in the Appendix.

The set of all lotteries that can be constructed over three outcomes is often geometrically represented as the probability triangle (see Machina, 1982). An individual's indifference curve shows all lotteries with the same expected utility. In other words, when choosing between two lotteries located on the same indifference curve, an individual chooses with probabilities 50%–50%. Under expected utility theory indifference curves are parallel straight lines.

Index $I_{♀}(x,y,z)\equiv[u_{♀}(x)-u_{♀}(y)]/[u_{♀}(z)-u_{♀}(x)]$ measures local risk aversion of an individual ♀ in the context of outcomes x, y, $z\in X$, $z\succsim x\succsim y$. Index $I_{♀}(x,y,z)$ is equal to the slope of ♀'s indifference curves in the probability triangle representing all lotteries over outcomes x, y, $z\in X$, $z\succsim x\succsim y$. Inequality (21) effectively states that the indifference curves of a less risk averse individual ♂ should never be steeper than the indifference curves of a more risk averse individual ♀ (see figure 1). Thus, in model (10) definition 2 leads to a familiar notion of risk aversion (*cf.* Machina, 1982).

[INSERT FIGURE 1 HERE]

The importance of proposition 3 for applied econometrics is difficult to underestimate. The notion of probabilistic risk aversion cannot be defined in the original strong utility (Fechner) model (6). Thus, model (6) cannot be used for estimating risk attitudes, which is one of its most common econometric applications. In contrast, the notion of probabilistic risk aversion is well-defined in model (10). In particular, this notion is the same as a standard definition of risk aversion for classical expected utility theory. Thus, not only model (10) can be used for estimating risk attitudes but it also produces the results that have standard microeconomic interpretation.

7. Conclusion

The main idea of strong utility (Fechner) model is quite intuitive. People do not automatically pick an alternative which maximizes their utility. Instead, they choose probabilistically. The higher the utility of a choice alternative, the more it is likely to be chosen. This simple intuition adds to the popularity of strong utility (Fechner) model. This paper proposes a modification of strong utility. The modified model has four clear advantages over the original model:

1) no violations of first order stochastic dominance,

2) superior goodness of fit to experimental data,

3) ability to rationalize systematic preference reversals,

4) a well-defined notion of probabilistic risk aversion.

Axiomatic characterization of the proposed model reveals an interesting kinship to the model of Fishburn (1978). Both models can be derived from the same system of axioms differing only in one arithmetic assumption. In this paper we assume that choice probabilities depend on differences in contextual probability equivalents. In contrast, the model of Fishburn (1978) assumes that they are driven by the ratio of contextual probability equivalents.

The proposed model can be applied not only in choice under risk but also in other microeconomic domains such as, for example, consumer choice. Consider the commodity space $\mathbb{R}^L$. Let $A \in \mathbb{R}^L$ be a commodity bundle (vector). For any two bundles A, $B \in \mathbb{R}^L$, a commodity bundle $A \vee B$ is defined as bundle $[\max\{A_1, B_1\}, ..., \max\{A_L, B_L\}]$ and a commodity bundle $A \wedge B$ is defined as bundle $[\min\{A_1, B_1\}, ..., \min\{A_L, B_L\}]$. Modified strong utility (Fechner) model postulates that a consumer chooses bundle A over bundle B with probability:

(22)
$$P(A,B) = F\left(\frac{U(A) - U(B)}{U(A \vee B) - U(A \wedge B)}\right), \qquad \text{if } U(A \vee B) \neq U(A \wedge B)$$

$$P(A,B) = 0.5, \qquad \text{if } U(A \vee B) = U(A \wedge B).$$

where F:$[-1,1] \rightarrow [0,1]$ is a non-decreasing function such that $F(v) + F(-v) = 1$ for all $v \in [-1,1]$, and $U: \mathbb{R}^L \rightarrow \mathbb{R}$ is consumer's utility function.

According to model (22), revealed choices are always monotone. Yet, if neither bundle dominates the other, a consumer chooses in a probabilistic manner. Thus, formula (22) can be applied for modeling variable consumer demand. This approach has a comparative advantage over the traditional random preference/utility approach. Formula (22) satisfies weak stochastic transitivity whereas random preference/utility leads to intransitive choice cycles akin to the Condorcet's paradox.

The model proposed in this paper can be also applied in game theory. In standard Nash equilibrium, players can randomize only in order to keep the opponent in the state of indifference. McKelvey and Palfrey (1995) developed the concept of quantal response equilibrium (QRE) based on strong utility

(Fechner) model.[7] In QRE players must not worry about keeping the opponent indifferent. Instead, they choose what is in their best interest (in a probabilistic manner) given their beliefs about opponent's actions. A similar solution concept for non-cooperative games can be developed based on the model proposed in this paper. The advantage of such solution concept is that players avoid choosing dominated strategies because they are bound to respect stochastic dominance.

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[7] A popular class of QRE is logit QRE, which is based on model (6) with error term ξ drawn from the logistic distribution. This specification of strong utility is sometimes called Luce's choice model (*cf.* Luce, 1959).

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Appendix

Proof of Proposition 3 Consider an arbitrary outcome $x \in X$ and a lottery $L \in \mathcal{L}$. We shall prove that $P_{♀}(x,L) \geq P_{♂}(x,L)$ if conditions of proposition 3 hold.

If outcome x is less preferred than the worst possible outcome in lottery L, then model (10) implies that $P_{♀}(x,L) = F_{♀}(-1)$ and $P_{♂}(x,L) = F_{♂}(-1)$. Since $F_{♀}(-1) = F_{♂}(-1)$, it must be the case that $P_{♀}(x,L) = P_{♂}(x,L)$. Similarly, if outcome x is more preferred than the best possible outcome in lottery L, then we have $P_{♀}(x,L) = F_{♀}(1) = F_{♂}(1) = P_{♂}(x,L)$. What remains is to consider the case when outcome x lies between the worst and the best possible outcome in lottery L.

If inequality (21) holds for all x, y, $z \in X$ such that $z \succsim x \succsim y$ then

(A1) $$\frac{\sum_{y \in X,\, x \succsim y} L(y) \cdot [u_{♀}(x) - u_{♀}(y)]}{\sum_{z \in X,\, z \succsim x} L(z) \cdot [u_{♀}(z) - u_{♀}(x)]} \geq \frac{\sum_{y \in X,\, x \succsim y} L(y) \cdot [u_{♂}(x) - u_{♂}(y)]}{\sum_{z \in X,\, z \succsim x} L(z) \cdot [u_{♂}(z) - u_{♂}(x)]}$$

If inequality (A1) holds then the following inequality holds as well:

(A2) $$\frac{\sum_{y \in X,\, x \succsim y} L(y) \cdot [u_{♀}(x) - u_{♀}(y)] - \sum_{z \in X,\, z \succsim x} L(z) \cdot [u_{♀}(z) - u_{♀}(x)]}{\sum_{y \in X,\, x \succsim y} L(y) \cdot [u_{♀}(x) - u_{♀}(y)] + \sum_{z \in X,\, z \succsim x} L(z) \cdot [u_{♀}(z) - u_{♀}(x)]} \geq$$
$$\geq \frac{\sum_{y \in X,\, x \succsim y} L(y) \cdot [u_{♂}(x) - u_{♂}(y)] - \sum_{z \in X,\, z \succsim x} L(z) \cdot [u_{♂}(z) - u_{♂}(x)]}{\sum_{y \in X,\, x \succsim y} L(y) \cdot [u_{♂}(x) - u_{♂}(y)] + \sum_{z \in X,\, z \succsim x} L(z) \cdot [u_{♂}(z) - u_{♂}(x)]}$$

Since $F_{♀}(v) = F_{♂}(v)$ for all $v \in [-1,1]$ and it is a non-decreasing function, we can rewrite inequality (A2) as

(A3) $$F_{♀}\left(\frac{u_{♀}(x) - \sum_{y \in X} L(y) \cdot u_{♀}(y)}{\sum_{y \in X,\, x \succsim y} L(y) \cdot [u_{♀}(x) - u_{♀}(y)] + \sum_{z \in X,\, z \succsim x} L(z) \cdot [u_{♀}(z) - u_{♀}(x)]}\right) \geq$$
$$\geq F_{♂}\left(\frac{u_{♂}(x) - \sum_{y \in X} L(y) \cdot u_{♂}(y)}{\sum_{y \in X,\, x \succsim y} L(y) \cdot [u_{♂}(x) - u_{♂}(y)] + \sum_{z \in X,\, z \succsim x} L(z) \cdot [u_{♂}(z) - u_{♂}(x)]}\right)$$

According to formula (10), the left hand side of (A3) is equal to $P_{♀}(x,L)$ and the right hand side of (A3) is equal to $P_{♂}(x,L)$. Thus, $P_{♀}(x,L) \geq P_{♂}(x,L)$. *Q.E.D.*

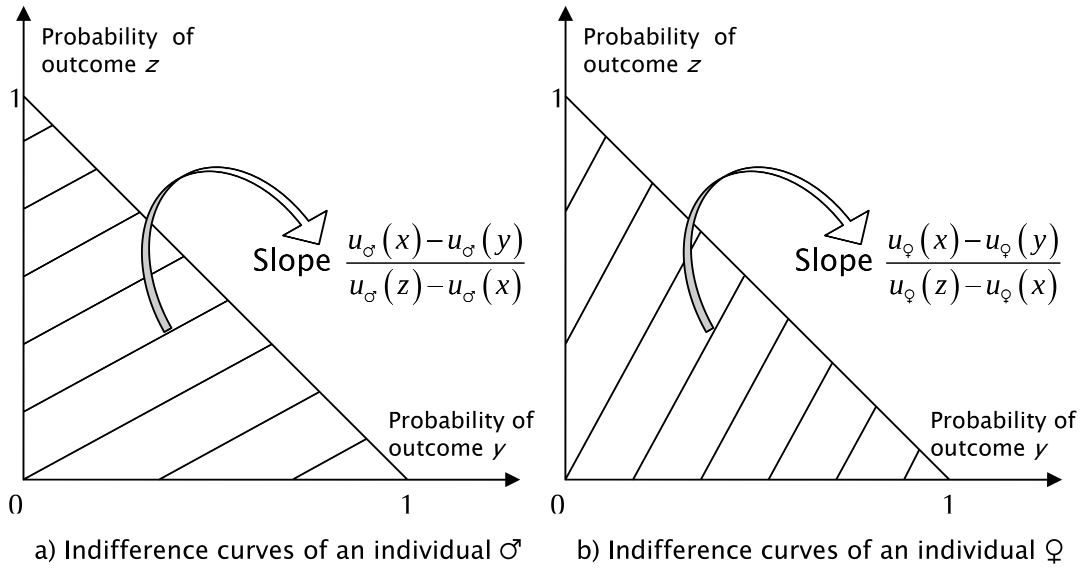

Figure 1 An individual ♀ is more risk averse than an individual ♂

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University of Innsbruck

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Pavlo Blavatskyy

Stronger Utility

Abstract
Empirical research often requires a method how to convert a deterministic economic theory into an econometric model. A popular method is to add a random error term on the utility scale. This method, however, violates stochastic dominance. A modification of this method is proposed to avoid violations of dominance. The modified model compares favorably to other existing models in terms of goodness of fit to experimental data. The modified model can rationalize the preference reversal phenomenon. An intuitive axiomatic characterization of the modified model is provided. Important microeconomic concept of risk aversion is well-defined in the modified model.

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