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Incentive schemes, private information and the double-edged role of competition for agents*

Christina E. Bannier[†], Eberhard Feess[‡], Natalie Packham[§] and Markus Walzl[¶]

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Abstract

This paper examines the effect of imperfect labor market competition on the efficiency of compensation schemes in a setting with moral hazard and risk-averse agents, who have private information on their productivity. Two vertically differentiated firms compete for agents by offering contracts with fixed and variable payments. The superior firm employs both agent types in equilibrium, but the competitive pressure exerted by the inferior firm has a strong impact on contract design: For high degrees of vertical differentiation, i.e. low competition, low-ability agents are under-incentivized and exert too little effort. For high degrees of competition, high-ability agents are over-incentivized and bear too much risk. For a range of intermediate degrees of competition, however, agents' private information has no impact and both contracts are second-best. Interim efficiency of the least-cost separating allocation in the inferior firm is a sufficient condition for equilibrium existence. If this is violated, there can only be equilibria where the inferior firm "overbids", i.e. where it would not break even when attracting both agent types. Adding horizontal differentiation allows for pure-strategy equilibria even when there would be no equilibrium without overbidding in the pure vertical model, but equilibria with overbidding fail to exist.

JEL Classification: D82, D86, J31, J33

Keywords: Incentive compensation, screening, imperfect labor market competition, vertical differentiation, horizontal differentiation, risk aversion

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1 Introduction

In competitive labor markets, compensation packages include variable pay not only to induce effort, but also to attract the most talented workers. With increasingly globalized markets, firms compete ever more fiercely for highly talented agents. Despite strongly growing salaries and bonuses, however, performance has not always followed and even slumped in many industries, most notably the financial industry (Bebchuk, Cohen and Spamann, 2010). Against this backdrop, our paper examines how the degree of labor market competition between heterogeneous firms affects the design and efficiency of compensation packages in order to derive both positive and normative results.

We consider a model where two vertically differentiated firms compete to employ a risk-averse agent. The agent's effort is unobservable and her productivity level, which is either high or low, is private information. In addition, productivity is firm-specific, and both agent types have higher productivity in one firm. Firms offer contracts that consist of a fixed wage and a share of the stochastic output in the form of a piece rate. In equilibrium, the more productive (good) firm employs both agent types, but equilibrium contracts depend crucially on the competitive pressure exerted by the less productive (bad) firm.

Our model yields two sets of results: Economic results and theoretical insights on existence and properties of screening equilibria with imperfect competition. Our main economic insight is that an intermediate degree of competitive pressure is optimal for incentivizing agents efficiently. More specifically, three regions need to be distinguished: If the productivity difference between the two firms is large, the good firm's equilibrium contract offers resemble monopsonistic screening. As the high-type agent has an incentive to imitate the low type in this region when both are held on their reservation utilities (which are, in equilibrium, endogenously determined by the bad firm's offers), the piece rate in the low type's contract is distorted downwards from the second-best efficient piece rate.[1] In this *quasi-monopsonistic region*, labor contracts for low-productivity agents are hence too low-powered and inefficiency shrinks with increasing competition.

For small productivity differences between the firms, contract inefficiency is similar to perfect competition. The high type's piece rate is distorted upwards from the second-best level in order to reduce the low type's imitation incentive. As a consequence, the inefficiency in this *quasi-competitive region* rises with increasing competition. However, there is a full range of intermediate degrees of competition (*second-best region*), for which both piece rates are second-best, so that private information on types does not induce a welfare loss. The reason is that second-best contracts differ across types due to agents' risk aversion and unequal productivity, so that these diverging second-best contracts fulfill the incentive-compatibility constraints of both types. Altogether, we hence find that the efficiency of compensation schemes is hump-shaped in the degree of vertical differentiation. This result extends findings by Bénabou and Tirole (2016) from horizontal to vertical differentiation and shows that risk aversion yields a plateau

[1] The second-best contract equilibrates the marginal losses from insufficient incentives and inefficient risk sharing.

rather than just one degree of (second-best) efficient competition.

The main set of theoretical insights from our analysis concerns existence and properties of pure-strategy equilibria. Supporting Rothschild and Stiglitz (1976), we show that whenever the proportion of high types is sufficiently small, the least-cost separating allocation is interim efficient. We demonstrate that interim efficiency in the bad firm is a sufficient condition for the existence of a pure-strategy equilibrium in which the bad firm offers both types their expected output. We furthermore prove that the condition becomes less restrictive when the productivity of the bad firm increases, but more restrictive with increasing risk aversion of agents. Both effects are driven by the fact that the bad firm's incentive to offer the low type more than her expected output (in order to decrease the upwards distortion in the high type's contract that is required for incentive compatibility) decreases in productivity, but increases in risk aversion.

Next, we show that, when the least-cost separating allocation is not interim efficient in the bad firm, then there exists no equilibrium in which the bad firm offers a contract menu where it would at least break even when attracting both types. The reason is that such a contract menu always induces a best response by the good firm that would allow the bad firm to attract both types with a cross-subsidy contract, which offers more than her expected output to the low type but less than her expected output to the high type. As a consequence, the only remaining candidate equilibrium contracts by the bad firm in this case offer the low type more than her output and the high type exactly her output. While such an *overbidding*-contract menu is weakly dominated as the bad firm would face losses with the low type and just break even with the high type, this strategy arises as the limit of undominated strategies, i.e., we consider limit admissible Nash equilibria, an extension of equilibria in weakly undominated strategies to infinite normal form games introduced by Simon and Stinchcombe (1995). Observe that the bad firm has no incentive to deviate from such a contract menu as long as both agent types are employed by the good firm, and if there is no other contract pair allowing to profitably attract at least one agent type. However, even such an overbidding equilibrium may fail to exist and, by contrast to equilibria where the bad firm offers an interim efficient least-cost allocation, existence may well be non-monotonic in the degree of vertical differentiation: Overbidding equilibria may subsist for low and for high levels of competitive pressure, but may fail to exist for intermediate levels (keeping all other parameters constant).

As a consequence of vertical differentiation, the good firm employs both agent types, while the bad firm is inactive in equilibrium. To see which of our economic and technical insights survive when considering a model where both firms are active in equilibrium, we introduce horizontal differentiation in addition to vertical differentiation by assuming that workers have preferences for which firm to work for. In this model, both firms employ a positive mass of workers if the degree of horizontal differentiation is sufficiently large compared to vertical differentiation. We show that it is again the less productive firm that is crucial for existence of equilibria. Interim efficiency of the least-cost separating allocation in the bad firm for a degree of horizontal differentiation of $t = 0$ is a sufficient, but no necessary condition for equilibrium existence. The reason is that the condition becomes less restrictive when the degree of horizontal differentiation t

increases. Thus, pure-strategy equilibria exist for $t > 0$ even in cases where they do not exist for $t = 0$. As in the basic model, the condition for equilibrium existence without overbidding becomes more restrictive when vertical differentiation increases. In this sense, horizontal and vertical differentiation have opposite effects on equilibrium existence. Equilibria with overbidding fail to exist in the extended model, however, as the bad firm would not break even and hence cannot be active in such equilibria. Apart from these technical insights, our applied economic results hold in the model with horizontal differentiation as well: Efficiency is still hump-shaped in the degree of competition, and a second-best efficient equilibrium exists for a whole region which now depends on both the degree of vertical and horizontal differentiation.

Our paper contributes to two bodies of literature: The literature on screening with competing principals, and the recent applied work on incentive contracts in labor markets. With regard to the former, Rothschild and Stiglitz (1976) show in their seminal work on perfectly competitive insurance markets with information asymmetry and common values[2] that existence of equilibrium in pure strategies requires that the least-cost separating allocation is interim efficient. In equilibrium, principals earn zero profits with either agent type. Both features carry over to the contracts offered by the bad firm in our model in any equilibrium without overbidding.

Subsequent research has addressed the equilibrium non-existence problem in models with perfect competition by allowing for mixed strategies (Dasgupta and Maskin, 1986) or by considering anticipatory or reactive strategies of principals, essentially changing the equilibrium concept (Wilson, 1977; Miyazaki, 1977; Riley, 1979). Later work has focused on extending the time structure of the model, allowing for dynamic interactions between competitors (Hellwig, 1987; Mimra and Wambach, 2011; Netzer and Scheuer, 2014; Handel, Hendel and Whinston, 2014), or on altering the characteristics of contracts or principals, e.g. by adding capacity constraints or nonexclusivity of contracts (Bisin and Gottardi, 1999; Inderst and Wambach, 2001; Schmidt-Mohr and Villas-Boas, 2008; Picard, 2014).[3]

In our model, the competitive pressure exerted by the bad firm defines the agents' outside options from the good firm's point of view. Thereby, the bad firm's equilibrium contract offers need to ensure that the best response offers by the good firm do not allow for a profitable deviation in the bad firm. Thus, the agent's outside options are determined by the productivity of the bad firm, and arise endogenously from the conditions that need to hold in equilibrium. This distinguishes our model from the literature that has studied the impact of exogenously introduced type-dependent reservation utilities (Lewis and Sappington (1989), Maggi and Rodriguez-Clare (1995) and Jullien (2000)). The latter paper also examines an application with competing principals. Accounting for such non-trivial participation constraints, Jullien (2000) shows that agents' informational rents are non-monotonic and may vanish for interior types. The corresponding

[2]For a discussion of agency problems with common values, where the agents' characteristics directly enter the principals' profit function, see Maskin and Tirole (1992). Pouyet, Salanié, and Salanié (2008) offer a concise explanation why the private values case leads to different results for competitive screening.

[3]For an overview of the screening and signalling literature, see Riley (2001), for a review of the more recent literature on screening in perfectly competitive markets, see also Mimra and Wambach (2014).

efficiency of contracts for intermediate types is similar to our results.

Aspects of screening under imperfect competition have also been considered in models of price discrimination.[4] Starting with the seminal work by Mussa and Rosen (1978) on monopolistic price discrimination, later work has introduced imperfect competition via both horizontal (Spulber, 1989; Schmidt-Mohr and Villas-Boas, 1999) and vertical differentiation (Stole, 1995) between firms. However, the main focus of these models is on the consequences of competition for efficiency rather than equilibrium existence.

The newer literature on price discrimination has considered a multitude of different modelling devices: Garrett, Gomes, and Maestri (2014) examine a competitive screening model where agents are heterogeneously informed about the offers available on the market. They find efficient quality provision only for the case of perfect competition, where the equilibrium converges to the Bertrand outcome. Attar, Mariotti, and Salanié (2014a) and Attar, Mariotti, and Salanié (2014b) study nonexclusive contracting such that each agent can contract with several principals. Attar, Mariotti, and Salanié (2014a) consider the insurance market and demonstrate that multiple contracting naturally emerges in a unique, constrained-efficient equilibrium.[5] Attar, Mariotti, and Salanié (2014b) show that nonexclusivity exacerbates the adverse selection problem in the Rothschild-Stiglitz world.

With regard to the second strand of the literature on contracting in labor markets, different aspects such as team work or multi tasking have recently been studied with various consequences for managerial incentives. Tymula (2014) analyzes a model where firms can employ teams of two agents with unobservable ability. Each of the two team members can invest in own tasks and in activities improving the output of her teammate. Under perfect competition, incentive contracts for good types are excessively high-powered.[6] This prevents bad types from imitating and establishes a separating equilibrium with assortative matching. Thanassoulis (2013) considers moral hazard with respect to effort and risk-taking in a competitive market equilibrium. Variable payments enhance effort and risk-taking incentives at the same time. As we do, he finds that fierce competition leads to excessively high-powered incentive contracts. However, there is no consideration of screening aspects in his model as managers learn their abilities only after contracting.

Garrett and Pavan (2012) and Garrett and Pavan (2015) study a dynamic moral hazard model where the quality of the match between firms and managers can change stochastically over time. They show that the optimal retention policy of a firm becomes more permissive with time and that managers' risk aversion requires lower powers of incentives with increasing tenure. Bannier, Feess, and Packham (2013) analyze the impact of screening contracts with imperfect labor market competition on risk-taking in portfolio management. Stronger competition among vertically differentiated firms

[4]See Stole (2007) for an overview of price discrimination in competitive settings.

[5]Interestingly, equilibrium features cross-subsidization of agent types due to the use of latent contracts (Arnott and Stiglitz, 1991) that are inactive on the equilibrium path but allow to deter cream-skimming deviations.

[6]Also with perfect competition in a multi-task model, Moen and Rosen (2005) demonstrate that incentives may be too high-powered, thereby distorting relative incentives.

leads to inefficiently high piece rates. As the positive effects of effort are ignored, social welfare is strictly decreasing in piece rates, and hence also in the degree of competition. Equilibrium existence is no concern in their model.

Most closely related to our work is the paper by Bénabou and Tirole (2016) who also consider imperfect labor market competition. In their model, horizontally differentiated firms compete in a Hotelling-framework for agents whose abilities are private information and who can perform two different tasks. While the first task is easily measurable, the second is not and contains elements of a public good. Besides abilities, agents also differ in their intrinsic motivation for performing the second task. As in our paper, the impact of labor market competition on social welfare is hump-shaped. For imperfect competition, Bénabou and Tirole (2016) restrict attention to risk-neutral agents, which explains why the second-best is achieved only at a single level of competition.[7] In contrast, we cover the full range of incomplete competition for risk-averse agents, so that our paper complements their analysis. Furthermore, Bénabou and Tirole (2016) restrict attention to cases where the least-cost separating allocation is interim efficient, and we prove that this is indeed a necessary condition for existence of equilibria in which both firms are active. This is an important difference to our basic model on the impact of competitive pressure exerted by less productive firms.

Altogether, this literature supports the prevalent view that competition for agents induces excessively high-powered incentive contracts for sought-after talents. While many papers focus on excessive risk-taking triggered by high-powered incentive contracts, our paper complements this literature by extending the classical incentive model with risk-averse agents and moral hazard to private information and imperfect competition. We find that unduly high-powered incentives that impose too much risk on agents are only obtained for high degrees of competition, and do not affect low-ability workers. In turn, low-ability agents will be under-incentivized for low degrees of labor competition.

The remainder of the paper is organized as follows: Section 2 presents the model. Section 3 derives the good firm's best-response function. Section 4 examines existence and characterization of equilibria, most notably the effect of competition on efficiency. Here, we differentiate between the cases where the least-cost separating allocation is interim efficient in the bad firm (Subsection 4.1) and where it is not (Subsection 4.2). In Section 5, we extend the model with pure vertical differentiation between firms by horizontal differentiation. We conclude in Section 6.

2 The model

Firms, agents and productivity. Two risk neutral firms $k \in \{G, B\}$ compete for a risk averse agent. The agent's ability type $i \in \{H, L\}$ is private information, and is

[7] Albeit in a very different framework, Biglaiser and Mezzetti (1993) also find that screening does not lead to distortions in case of intermediate competition. However, competition is defined with respect to the agent types, the low-ability type may choose inefficiently high effort, and a pooling equilibrium arises for intermediate types.

H (high) with probability α and L (low) with probability $1-\alpha$. The agent's effort e is unobservable, and the output of agent i when working for firm k is $\beta_k \theta_i e + \sigma Z$ where $\sigma > 0$, Z is a standard normally distributed random variable, and $\beta_k \in [0,1]$ and $\theta_i > 0$ capture the productivity relative to the firm and the agent type, respectively. We assume that $\theta_H > \theta_L$ and $\beta_G = 1 > \beta_B = \beta$. Thus, expected output depends on the agent's type via θ_i, her effort e, and the firm she works for via β_k. The agent's risk aversion is represented by an exponential utility function with constant coefficient of absolute risk aversion ρ. The agent receives a payoff P_i^k, and e^2 is the effort cost that she faces when exerting effort e, so that her utility is $U(P_i^k - e^2) = 1 - \mathbf{e}^{-\rho(P_i^k - e^2)}$. We normalize the agent's exogenous outside option $\overline{U}$ to zero.

Competition for agents. Firms compete for the agent by simultaneously offering take-it-or-leave-it screening contracts $(F_i, w_i) \in \mathbb{R} \times [0,1]$, where F_i is a fixed wage and w_i is a piece rate for type $i \in \{L, H\}$.[8]

The parameter β introduced above describes a simple form of vertical differentiation between the firms: Both agent types are more productive in the good firm than in the bad firm. As such, β captures the basic ingredient of competition for agents between the two firms. In particular, this modelling choice gives rise to a very simple representation of the two extreme cases of competition: We have perfect competition if $\beta = 1$, while we are in the simple case of monopolistic screening for $\beta = 0$.

Payoffs. Given the firms' compensation schemes, the agent's payoff P_i^k is given by

$$P_i^k := P_i^k(F, w, e) = F + w\left(\beta_k \theta_i e + \sigma Z\right).$$

As the error term Z is normally distributed, it follows from the moment-generating function of the normal distribution that the agent's expected utility is

$$\mathbb{E}\left[U(P_i^k - e^2)\right] = 1 - \mathbf{e}^{-\rho(F + w\beta_k\theta_i e - e^2 - \frac{\rho}{2}w^2\sigma^2)}.$$

Maximizing the agent's expected utility coincides with maximizing her certainty equivalent,

$$U_i^k(F, w, e) := F + w\beta_k\theta_i e - e^2 - \frac{\rho}{2}w^2\sigma^2. \tag{1}$$

The agent's effort choice is given by

$$e_i^k := e_i^k(w) = \operatorname*{argmax}_{e \geq 0}\left\{F + w\beta_k\theta_i e - e^2 - \frac{\rho}{2}w^2\sigma^2\right\} = \frac{1}{2}w\beta_k\theta_i.$$

[8] The optimality of linear sharing rules has been demonstrated by Holmstrom and Milgrom (1987) where an agent controls the drift $\mu(t)$ in the time interval $[0,1]$ of the process $\mathrm{d}Z = \mu(t)\mathrm{d}t + \sigma \mathrm{d}B$ with B a standard Brownian motion. The agent is risk averse with constant absolute risk aversion. The principal observes the path $(Z_t)_{0 \leq t \leq 1}$ and compensation takes place at time 1 in form of a sharing rule $s((Z_t)_{0 \leq t \leq 1})$ with the sharing rule agreed at time 0. In this setup, the optimal drift choice is a constant drift μ and the optimal sharing rule is a linear function of Z_1, see Theorem 7 of Holmstrom and Milgrom (1987). Sung (2005) shows that the optimal sharing rule remains linear in a setup with moral hazard and adverse selection, see Section 2 and Theorem A.2 of Sung (2005). Packham (2015) proves that the linear sharing rule is also optimal in our setting where reservation utilities are type-dependent.

Inserting into (1) and simplifying yields

$$U_i^k(F,w) := U_i^k\left(F, w, \frac{1}{2}w\beta_k\theta_i\right) = F + \frac{w^2}{4}(\beta_k^2\theta_i^2 - 2\rho\sigma^2) \tag{2}$$

as the agent's certainty equivalent.[9] We define $\widehat{U}_i^k$ as the maximum certainty equivalent agent i can get from firm k, that is,

$$\widehat{U}_i^k := \max_{(F,w)\in\mathbf{\Omega}^k} U_i^k(F,w),$$

where $\mathbf{\Omega}^k$ denotes the set of contracts offered by principal k. Without loss of generality, we introduce the tie-breaking rule that both types accept the good firm's offer if $\widehat{U}_i^G = \widehat{U}_i^B$.

Note that the marginal utility of the piece rate is higher for the high type, that is, the single-crossing property holds:

$$\frac{\partial^2 U_i^k}{\partial w \partial \theta_i} = w\beta_k^2\theta_i > 0. \tag{3}$$

Finally, firm k's expected profit from agent i is

$$\Pi_i^k(F,w) := (1-w)\beta_k\theta_i e_i^k - F = \frac{1}{2}(1-w)w\beta_k^2\theta_i^2 - F. \tag{4}$$

Sequence of events. We consider the following game:

Stage 0: Nature chooses the agent's type which becomes private information.

Stage 1: Firms simultaneously offer take-it-or-leave-it contracts to the agent.

Stage 2: Depending on her type, the agent chooses her utility-maximizing contract and her effort.

Stage 3: Profits and payments are realized.

Complete information and the second-best piece rate. For later reference let us first consider the case without private information on types. In this case, each firm faces the usual trade-off between risk allocation and incentives and implements two second-best piece rates, which equilibrate the losses from inefficiently low effort and from insufficient risk-sharing at the margin. For each type, firm k maximizes profits as given in (4) subject to the following binding participation constraint *(PC)* defined by the maximum utility $\widehat{U}_i^{\overline{k}}$ type i could get in the competing firm $\overline{k} \neq k$:

$$F + \frac{w^2}{4}(\beta_k^2\theta_i^2 - 2\rho\sigma^2) = \widehat{U}_i^{\overline{k}}.$$

[9] In the following, we use the terms certainty equivalent and expected utility interchangeably.

After substituting for F and simplifying, we obtain

$$\Pi_i^k(w) = \frac{1}{2} w \left(\beta_k^2 \theta_i^2 - \frac{w \beta_k^2 \theta_i^2}{2} - w \rho \sigma^2 \right) - \widehat{U}_i^{\bar{k}},$$

and maximizing Π_i^k with respect to w yields the second-best piece rate

$$w_i^{k,sb} = \frac{\beta_k^2 \theta_i^2}{\beta_k^2 \theta_i^2 + 2\rho\sigma^2} \ . \tag{5}$$

Observe that $w_i^{k,sb}$ increases in the agent's type-dependent productivity θ_i and in the firm-dependent productivity β_k and decreases in the risk-aversion parameter ρ.

Welfare. In our model, different degrees of competition imply different allocations of the generated surplus between firms and workers. For welfare comparisons we consider the total surplus

$$W = \sum_{k=G,B} (\Pi^k + \alpha \widehat{U}_H^k + (1-\alpha) \widehat{U}_L^k),$$

i.e., we apply the concept of Kaldor-Hicks efficiency and consider an outcome as more efficient if those who are made better off could compensate those who are made worse off, so that a Pareto improvement could be achieved. We shall see that in any equilibrium, both worker types are employed by the good firm, and at least one piece rate is second-best. Thus, our welfare criterion effectively boils down to the degree of distortion in the piece rate that is not second-best.

3 The good firm's best response

In this section, we derive basic properties of a firm's best response *if* this firm hires both types of agents. As the good firm indeed employs both types of agents in equilibrium (see Lemma 3 below) we restrict ourselves to the good firm's best response in this section and continue with the bad firm's equilibrium behavior in Section 4. To simplify the exposition, we assume that workers choose the good firm when they are indifferent between the offers of both firms. Since we restrict attention to $\beta < 1$, this is without loss of generality as the good firm would marginally outbid the bad firm anyway due to its productivity advantage.

Assuming that the good firm wants to attract both agent types, its best-response function is a solution to[10]

$$\begin{aligned} \max_{F_H, w_H, F_L, w_L} \ & \Pi(F_H, w_H, F_L, w_L) \\ & = \alpha \left(\frac{1}{2} (1 - w_H) \, w_H \theta_H^2 - F_H \right) + (1-\alpha) \left(\frac{1}{2} (1 - w_L) \, w_L \theta_L^2 - F_L \right) \end{aligned} \tag{6}$$

[10] As this section addresses the good firm's best response, we omit the firm's superscript.

subject to the following constraints:

$$U_H(F_H, w_H) \geq \widehat{U}_H^B, \qquad \textit{(PCH)},$$

$$U_L(F_L, w_L) \geq \widehat{U}_L^B, \qquad \textit{(PCL)},$$

$$U_H(F_H, w_H) \geq U_H(F_L, w_L), \qquad \textit{(ICCH)},$$

$$U_L(F_L, w_L) \geq U_L(F_H, w_H) \qquad \textit{(ICCL)}.$$

Here, $\widehat{U}_i^B$ is the maximum utility agent type i can gain from the bad firm's contract offers. From the good firm's perspective, $\widehat{U}_i^B$ is agent i's reservation utility. As usual, $\widehat{U}_i^B$ is exogenous in the good firm's best response and endogenously determined in equilibrium.

Solutions to (6) have a concise structure.

Lemma 1. *Let $(F_H^*, w_H^*), (F_L^*, w_L^*)$ be a solution to (6). Then,*

1. *$w_H^* \geq w_H^{sb}$ and $w_L^* \leq w_L^{sb}$;*
2. *If $w_L^* < w_L^{sb}$, then:* (i) *$w_H^* = w_H^{sb}$;* (ii) (ICCH) *and* (PCL) *are binding, while* (iii) (ICCL) *is non-binding;*
3. *If $w_H^* > w_H^{sb}$, then:* (i) *$w_L^* = w_L^{sb}$;* (ii) (ICCL) *and* (PCH) *are binding, while* (iii) (ICCH) *is non-binding.*

All proofs are in the Appendix.

According to the Lemma, at most one piece rate will be distorted because at most one agent type has an imitation incentive when two second-best piece rates are offered. When the high type has the imitation incentive, then the low type's piece rate is distorted, her incentive compatibility constraint (ICCL) is non-binding and her participation constraint (PCL) is binding. The high type's piece rate is second-best efficient ("no distortion at the top") and her incentive compatibility constraint (ICCH) is binding (Part 2 of the Lemma). These features are known from monopsonistic screening. Part 3 states analogous features known from competitive screening where the low type has an imitation incentive resulting in a binding (ICCL) and (PCH), and a second-best piece rate for the low type ("no distortion at the bottom").

Hence, solutions to (6) are either *quasi-monopsonistic* with a distorted piece rate for low types, *quasi-competitive* with a distorted piece rate for high types, or have *second-best* piece rates for both types. The following Proposition shows that the optimality of each of these contract types depends in a monotone fashion on the *utility spread* $\Delta\widehat{U}^B := \widehat{U}_H^B - \widehat{U}_L^B$ offered by the bad firm.

Proposition 1. *Let $(F_H^*, w_H^*), (F_L^*, w_L^*)$ be a solution to (6). Then, there exist $\Delta\widehat{U}_{QM}^B$, $\Delta\widehat{U}_{QC}^B \in \mathbb{R}$ with $\Delta\widehat{U}_{QM}^B < \Delta\widehat{U}_{QC}^B$ such that:*

Region 1 (Quasi-monopsonistic, QM): *If* $\Delta\widehat{U}^B < \Delta\widehat{U}^B_{QM}$*, the high type's piece rate is second-best,* $w_H^* = w_H^{sb}$*, and* (PCL) *and* (ICCH) *are binding. The piece rate for the low type is below second-best,* $w_L^* < w_L^{sb}$*, and determined by:*

(a) *the first-order condition of the good firm's maximization problem, in which case the high type's participation constraint,* (PCH)*, is non-binding;*

(b) *the binding* (PCH) *otherwise.*

Both $\Delta\widehat{U}^B$ *and* w_L^* *are greater in Region QM(b) than in Region QM(a), and* w_L^* *is strictly increasing in* $\Delta\widehat{U}^B$ *in Region QM(b). Social welfare is increasing in* $\Delta\widehat{U}^B$*.*

Region 2 (Second-best, SB): *If* $\Delta\widehat{U}^B \in \left[\Delta\widehat{U}^B_{QM}, \Delta\widehat{U}^B_{QC}\right]$*, both piece rates are second-best.*

Region 3 (Quasi-competitive, QC): *If* $\Delta\widehat{U}^B > \Delta\widehat{U}^B_{QC}$*, the low type's piece rate is second-best,* $w_L^* = w_L^{sb}$*, and* (PCH) *and* (ICCL) *are binding. The piece rate for the high type is above second-best,* $w_H^* > w_H^{sb}$*, and determined by:*

(a) *the first-order condition of the good firm's maximization problem, in which case the low type's participation constraint,* (PCL)*, is non-binding;*

(b) *the binding* (PCL) *otherwise.*

Both $\Delta\widehat{U}^B$ *and* w_H^* *are greater in Region QC(a) than in Region QC(b) and* w_H^* *is strictly increasing in* $\Delta\widehat{U}^B$ *in Region QC(b). Social welfare is decreasing in* $\Delta\widehat{U}^B$*.*

For a low utility spread (i.e., $\Delta\widehat{U}^B < \Delta\widehat{U}^B_{QM}$), the best response is similar to optimal contracts in a monopsony: The high type has an imitation incentive and the good firm tries to equilibrate the marginal loss from the distortion in the low type's piece rate with the marginal reduction in the high type's information rent. For $\Delta\widehat{U}^B$ sufficiently close to zero (i.e., close to the monopsony case), the high type receives an information rent that is decreasing in her reservation utility. As $\Delta\widehat{U}^B$ increases, the information rent and thereby the need to distort the low type's piece rate for rent reduction diminishes. As a result, w_L^* (and social welfare) is increasing in $\Delta\widehat{U}^B$ (see Region QM(a)). If $\Delta\widehat{U}^B$ is so large that the high type does not receive an information rent (i.e., (PCH) is binding), there is no point in further reducing her imitation incentive, and $w_L < w_L^{sb}$ is determined by the binding (PCH) instead (Region QM(b)).[11]

For a high utility spread (i.e., $\Delta\widehat{U}^B > \Delta\widehat{U}^B_{QC}$), the best response is similar to optimal contracts under perfect competition: With a high utility spread offered by the bad firm, the low type has an imitation incentive whenever the good firm attracts both types. In order to reduce the low type's information rent, the contract for the high type is

[11] Any piece rate below would reduce the efficiency of the low type's contract, but would not allow to offer a lower overall remuneration to the high type. Consequently, the piece rate for the low type is higher in Region QM(b) than in Region QM(a).

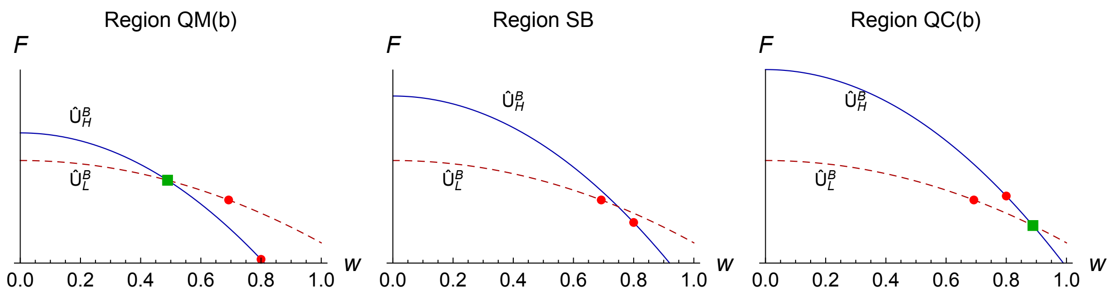

Figure 1: Best responses of good firm. In each graph, the solid (dashed) line denotes the high (low) type's indifference curve at the reservation level $\widehat{U}_H^B$ ($\widehat{U}_L^B$) for different combinations of fixed wages and piece rates. Points denote second-best contracts. In region QM(b), the high type has an imitation incentive when two second-best piece rates are offered as she would reach an indifference curve to the northeast when choosing the contract designed for the low type. In order to prevent imitation, the low type's piece rate is decreased to make *(ICCH)* binding; this is depicted by the rectangle. Likewise, in region QC(b), the low type has an imitation incentive, and the high type's contract is distorted to make *(ICCL)* binding (rectangle).

inefficiently high-powered. As the the utility spread increases, the imitation incentive becomes larger and the corresponding distortion of the high type's piece rate (weakly) increases (and social welfare decreases – see Region QC(a)). If the efficiency loss due to distorted piece rates for the high type becomes so large that an information rent is paid to low types (i.e., (PCL) is not binding), rents for the low type (but not piece-rate distortions) are increasing in $\Delta\widehat{U}^B$ (see Region QC(b)).

For intermediate values of $\Delta\widehat{U}^B$ contracts are second best. Inefficiencies arise only from moral hazard and risk aversion, and incomplete information poses no further constraints. Neither agent type can benefit from imitating the other when both are held on their reservation utilities and both piece rates are second-best efficient. The existence of a whole *second-best region* instead of just one utility level $\Delta\widehat{U}^B$ where this is the case follows from the fact that the agents' second-best piece rates differ as the trade-off between effort incentives and risk aversion is type-specific. Figure 1 illustrates the three different regions.

For risk neutral agents (i.e., $\rho = 0$), second-best piece rates are first best (i.e., $w_i^k = w_i^{k,fb} = 1$). With private information, the good type's piece rate is still distorted upwards for high $\Delta\widehat{U}^B$ while the low agent type's piece rate is distorted downwards for low $\Delta\widehat{U}^B$. The region where private information does not lead to a welfare reduction collapses to one point, namely where $\Delta\widehat{U}^B = \dfrac{1}{4}\left(\theta_H^2 - \theta_H^2\right)$. Thus, the existence of a whole region of (intermediate) degrees of competition, which are immune to adverse selection, can be attributed to agents' risk aversion.

4 Existence and characterization of equilibria

4.1 The least-cost separating allocation is interim efficient

As a starting point for the equilibrium analysis, we consider a contract menu by the bad firm where both types are offered their expected output, so that the firm breaks even when attracting both types. In this case, the low type has an imitation incentive, so that her piece rate is second-best and her (ICC) is binding, while the high type's piece rate is upwards distorted. We refer to this contract menu as the *bad firm's least-cost separating allocation (LCS)* (Bénabou and Tirole (2016)), and we show that, similar to the Rothschild-Stiglitz-model of perfect competition, this contract menu by the bad firm is part of an equilibrium if and only if the LCS is interim efficient.

Definition 1. An incentive compatible contract menu $(F_i^*, w_i^*)_{i=H,L}$ offered by firm k with profit $\Pi^{k,*} \geq 0$ is **interim efficient (IE)** if there is no other incentive-compatible menu $(F_i, w_i)_{i=H,L}$ with profit Π^k that Pareto dominates it: $U_i \geq U_i^*$ for $i \in \{H, L\}$ and $\Pi^k \geq \Pi^{k,*}$ with at least one inequality being strict.

To see the importance of IE of the LCS allocation, suppose that the bad firm offers both types their expected output and that both (PCL) and (PCH) are binding in the good firm's best response. In case LCS is not IE, the bad firm can profitably deviate by offering more than her expected output to the low type, which allows to reduce her imitation incentive and the corresponding upwards distortion in the high type's piece rate. As this new contract menu is Pareto dominant, the increase in the high type's surplus outweighs the loss from the low type. The higher the percentage of low types (i.e., the smaller α) the larger the firm's cost from offering more to low types and the smaller the gain from reducing distortions for high types. Thus, IE of LCS requires that α is sufficiently low.

To derive the critical proportion of high types, α_{LCS}, that just ensures IE of LCS in the bad firm, denote the expected output of type i in firm k in response to a piece rate w_i^k as $v_i^k(w_i^k) = \frac{1}{2}\beta_k^2\theta_i^2 w_i^k(1 - w_i^k) + \frac{(w_i^k)^2}{4}(\beta_k^2\theta_i^2 - 2\rho\sigma^2)$. A menu $(F_i^*, w_i^*)_{i=H,L}$ is interim efficient if and only if the loss from relaxing the low type's incentive compatibility constraint by a marginal decrease of w_H (i.e., $(1 - \alpha)\frac{w_H^k}{2}\beta_k^2(\theta_H^2 - \theta_L^2)$) exceeds the gain from an output increase due to this marginal reduction of the high type's piece rate (i.e., $-\alpha\frac{dv_H^k(w_H^k)}{dw_H^k}$)). Hence, α has to be sufficiently small to ensure

$$\alpha\frac{dv_H^k(w_H^k)}{dw_H^k} + (1 - \alpha)\frac{w_H^k}{2}\beta_k^2(\theta_H^2 - \theta_L^2) \geq 0. \tag{7}$$

Denote by α_{LCS} the α that solves (7) with equality for $k = B$.

Proposition 2. (i) *The bad firm's least-cost separating allocation $(F_i^*, w_i^*)_{i=H,L}$ is interim efficient if and only if $\alpha \leq \alpha_{LCS}$;* (ii) *α_{LCS} is decreasing in ρ;* (iii) *α_{LCS} is increasing in β.*

The intuition for the impact of α on interim efficiency as depicted by *Part (i)* has been discussed above and is known from screening with perfect competition. *Part (ii)* expresses that the condition for IE of LCS becomes more restrictive when the degree of risk aversion increases. The higher the degree of risk aversion, the higher is the low type's marginal utility from getting a larger fixed salary (recall that her piece rate is second-best anyway) compared to the benefit from the larger piece rate when imitating the high type. Thus, her benefit when substituting variable with fixed payments increases in her degree of risk aversion, and the (less desired) variable component is larger in the high type's contract. Therefore, offering slightly more than expected output to the low type reduces her imitation incentive to a larger degree when risk aversion is high, which in turn allows for a larger reduction in the upwards distortion of the high type's piece rate. Higher risk aversion thus makes it more profitable for the firm to offer the low type more than her expected output, so that the condition for IE of LCS becomes more restrictive when risk aversion increases. In fact, LCS is always interim efficient for risk-neutral agents, i.e., $\alpha_{LCS} = 1$ when $\rho = 0$.

The intuition for *Part (iii)* proceeds along the same lines: The lower the productivity level β of the bad firm, the higher is the marginal utility an agent gains from getting a higher fixed payment compared to a higher variable payment. Similar to the impact of ρ, offering the low type more than her expected output is more profitable when β is low, since even a small increase in the low type's fixed salary leads to a strong reduction in her imitation incentive. Thus, the condition of interim efficiency becomes more restrictive. As $\beta = 1$ in the good firm, this implies that the condition for IE of LCS is more restrictive in the bad than in the good firm, i.e. $\alpha \leq \alpha_{LCS}(\beta)$ in the bad firm is sufficient, but not necessary for IE of LCS in the good firm.

Proposition 3. *If the LCS is interim efficient, then the LCS offered by the bad firm and the best response by the good firm as in Proposition 1 constitute an equilibrium.*

To see why offering both types their expected output by the bad firm constitutes an equilibrium when LCS is IE, note first that such a contract menu uniquely determines $\Delta\widehat{U}^B$, the spread in the two types' reservation utilities from the good firm's perspective. From Proposition 1, we know that three regions for the good firm's best-response function need to be distinguished. For each of those regions, we now examine whether the bad firm has a profitable deviation. Recall that it is always the low type who has the imitation incentive in the bad firm, while this is the case for the good firm only in the quasi-competitive region.

Quasi-monopsonistic region 1: In region QM, (PCL) is always binding in the good firm's best response. In region QM(b), (PCH) is binding as well. By definition of IE of LCS, the bad firm then has no profitable deviation from offering both types their output as any other contract menu would be Pareto inferior and thus lead to losses when attracting any type. In region QM(a), where (PCH) is non-binding, competitive pressure exerted by the bad firm is so low that the high type's utility from the good firm is given by equilibrating the marginal efficiency loss from the low type's contract with the marginal reduction in the high type's information rent. This is just as with monopsonistic screening when the difference in type-dependent reservation levels of utility is small. Again, the bad firm has no profitable deviation.[12] Note that the equilibrium is not unique as the bad firm cannot profitably deviate as long as the good firm's best response is in QM(a). Thus, if LCS is interim efficient in the bad firm and $\Delta\widehat{U}^B < \Delta\widehat{U}^B_{QM}$ holds in the LCS allocation, then there exists an equilibrium in region QM where the bad firm offers both types their expected output.

Second-best region 2: When the good firm's best response is in the second-best region, then (PCH) and (PCL) are both binding due to the fact that no type has an imitation incentive in the good firm. Thus, the argument provided for region QM(b) applies and a contract menu where both types get their expected output in the bad firm constitutes an equilibrium.

Quasi-competitive region 3: We now turn to the most interesting case where competitive pressure exerted by the bad firm is so large that it is the low type who has the imitation incentive in the good firm. Recall from Proposition 1 that (PCL) and (PCH) are both binding in region QC(b). Thus, IE of LCS again ensures that the bad firm has no profitable deviation. Next, assume *hypothetically* that the good firm's best response is in region QC(a) where *(PCL)* is non-binding. Such a best response by the good firm cannot be part of an equilibrium, because the bad firm has a profitable deviation: As (PCL) is non-binding, the bad firm can offer more to the low type without attracting her; thereby reducing the low type's imitation incentive and the necessary upwards distortion in the high type's piece rate. This efficiency gain can then be used to increase the high type's utility and the bad firm's profit at the same time. Thus, the bad firm has a profitable deviation whenever the good firm's best response is in region QC(a).

This case is helpful for later reference, but only hypothetical if LCS is IE: When LCS is IE in the bad firm and the bad firm offers both types their expected output, then the good firm's best response is not in region QC(a). To see this, recall from Proposition 2 *(iii)* that IE of LCS in the bad firm implies IE of LCS in the good firm. So *if* the good firm offers both agent types their expected output, it does not have an incentive to offer the low type more in order to reduce piece-rate distortions for the high type and (PCL) is binding. But offering the expected output is only a best reply under perfect competition (i.e., $\beta = 1$). For the LCS in the bad firm $\Delta\widehat{U}^B$ is increasing in β such that $\Delta\widehat{U}^B$ is larger for $\beta = 1$ than for $\beta < 1$. But as $\Delta\widehat{U}^B$ is larger in region QC(a) than in region QC(b) (see Proposition 1), the good firm's best response for $\beta < 1$ has to be in QC(b) and (PCL) is indeed binding.

For the comparative statics it suffices to observe that if the bad firm offers each type her output, $\Delta\widehat{U}^B$ is increasing in β. The larger β, the larger the utility spread offered by the bad firm and the more competitive is the best response of the good firm as specified in Proposition 1.

Proposition 4. *Suppose LCS is IE in an open neighborhood of β. Then $\dfrac{\partial \Delta\widehat{U}^B}{\partial \beta} \geq 0$. Moreover, whenever LCS is IE for β_0, β_1 with $\beta_0 < \beta_1$, then $\Delta\widehat{U}^B(\beta_0) \leq \Delta\widehat{U}^B(\beta_1)$.*

[12] The bad firm could offer more than expected output to the high type without attracting her, but this would only increase the low type's imitation incentive.

The intuition why $\Delta\widehat{U}^B$ is increasing in β is that a higher productivity also increases the difference in the two agent types' productivity, $\beta\left(\theta_H^B-\theta_L^B\right)$, and thereby also their utility difference when both are offered their expected output.[13] Together with Propositions 1 and 3 it follows directly that if LCS is IE at β, then there is a pure strategy equilibrium $(F_i^{k,*}, w_i^{k,*})_{i=H,L}$ such that the bad firm offers each agent type her output and *(i)* $w_H^G = w_H^{G,sb}$, $\frac{dw_L^G}{d\beta} > 0$, and welfare is increasing in β if the good firm's best response is in region QM(b); *(ii)* $w_H^G = w_H^{G,sb}$, $w_L^G = w_L^{G,sb}$, and welfare is independent of β if the good firm's best response is in region *SB*, *(iii)* $w_L^G = w_L^{G,sb}$, $\frac{dw_H^G}{d\beta} > 0$, and welfare is decreasing in β if the good firm's best response is in region QC. If β is sufficiently close to zero such that the good firm's best response is in region QM(a), the piece-rate distortion (and hence social welfare) is independent of the bad firm's behavior (and β).

4.2 The least-cost separating allocation is *not* interim efficient

We have shown that a pure-strategy equilibrium in which the bad firm would break even when attracting both types exists when LCS in the bad firm is IE. Now, we extend to the case where LCS is not IE ($\alpha > \alpha_{LCS}$). We exclude that the bad firm offers *both* types more than their output, i.e., the case where it would face losses when attracting just one or both of them. If the productivity difference between the two firms is sufficiently large, the good firm would still outbid the bad firm, which would thus have no incentive to deviate. Such a contract menu by the bad firm, however, is implausible as it is weakly dominated. It resembles the simplest case of a Bertrand duopoly with constant marginal costs, where the bad firm could offer any price below its own and above the competitor's marginal costs.

We do not exclude, however, the case where the bad firm offers the low type more than her expected output and the high type exactly her expected output. Such a contract menu – which we will refer to as *overbidding* – arises as the limit of undominated strategies because the bad firm could earn positive profits when offering the high type an arbitrarily smaller payoff than her expected output and when attracting this type only. Again, this is similar to Bertrand competition where charging a price equal to marginal costs is weakly dominated, but nevertheless the limit of undominated strategies (and part of the unique equilibrium in pure strategies).

We hence follow Simon and Stinchcombe (1995) and restrict ourselves to limit admissible strategies.[14]

Assumption 1. *Weakly dominated strategies are excluded, except those that are limits of undominated strategies.*

[13] The proof of the Proposition demonstrates that this direct effect is not altered by the indirect impact of β on optimal piece rates.

[14] Simon and Stinchcombe (1995) use the same criterion and call the resulting equilibria *limit admissible.* Limit admissibility is required for infinite games; otherwise one could just exclude all weakly dominated strategies.

Next, recall that in region QM(a) the competitive pressure exerted by the bad firm is so small that the downwards distortion in the low type's piece rate in the good firm's best response is determined as in monopsonistic screening, i.e., the good firm offers the high type more than her reservation utility in order to reduce her imitation incentive (non-binding *(PCH)*). For this case of particularly low competitive pressure, existence of equilibrium is no concern,[15] so that we subsequently ignore this case to streamline the analysis. We then make use of the following Lemma:

Lemma 2. *In equilibrium, if* (PCH) *is binding, then (i) the bad firm offers the high type her expected output; (ii) (PCL) is binding; (iii) the bad firm offers the low type at least her expected output.*

To explain, consider first that the bad firm has a profitable deviation if it offers less than expected output to the high type (*part (i)*): If *(PCL)* is non-binding, the bad firm can simply offer slightly more to the high type. If *(PCL)* is binding (which is the case in equilibrium, see *part (ii)*), the bad firm can offer the high type more (but still less than expected output) and avoids the low type's imitation by slightly increasing the high type's piece rate. Such a profitable deviation exists for all contract menus where the bad firm would earn positive profits when attracting both types. In equilibrium, the bad firm thus needs to compete as fiercely as possible.[16] Next, if *(PCL)* were non-binding in the good firm's best response, then the bad firm could offer more to the low type without attracting her. This would allow to reduce the upwards distortion in the high type's piece rate, and the corresponding efficiency gain would permit to profitably attract the high type. This explains *part (ii)* of the Lemma. *Part (iii)* expresses that there may be overbidding equilibria where the bad firm offers the high type her expected output and the low type more than her expected output.

As both agent types get at least their expected output from the bad firm, a contract menu entailing cross-subsidies cannot be part of an equilibrium: In a cross-subsidy contract, the part of the high type's output not offered yet could always be used to attract the high type and to gain positive profits at the same time, while avoiding the bad type's imitation incentive by increasing the high type's piece rate, thereby leaving the low type to the good firm. At the same time, cross-subsidy menus destroy a potential equilibrium where both types get their expected output if LCS is not IE. Thus, IE of LCS is not only a sufficient, but also a necessary condition for an equilibrium in which the bad firm would break even when attracting both types:[17]

Corollary 1. *In a pure-strategy equilibrium with binding* (PCH)*, there is no overbidding if and only if LCS is IE.*

[15]In an equilibrium with a non-binding *(PCH)*, the bad firm's behaviour is not uniquely specified. However, the contract pair where each type receives her expected output is contained in the set of best responses, even when LCS is not IE, as the bad firm has no deviation to profitably attract the high type within QM(a). If such a deviation exists, then it is because *(PCH)* is binding, so it is outside of Region QM(a).

[16]Observe that offering the high type more than her expected output is weakly dominated as it is always the low type who has the imitation incentive in the bad firm.

[17]Again, this neglects the case where β is so low that (PCH) in the good firm's best response is non-binding.

One final aspect needs to be considered for the discussion of equilibrium existence in case LCS is not IE in the bad firm, which concerns the good firm's behavior: In many monopolistic screening models, the firm offers just a single contract in order to eliminate the high type's information rent when the frequency of low types, $1-\alpha$, is below a certain threshold (e.g. Section 2.2 of Salanié (2005)). This is not the case in our model:

Lemma 3. *In any pure-strategy equilibrium, the good firm hires both agent types.*

The reason for Lemma 3 is as follows: As $(1-\alpha) \to 0$, the piece rate w_L in the low type's contract converges to 0, and so does hence the high type's information rent since the two types' utilities are the same for $w_L = 0$. Thus, offering two contracts or just one contract yields identical profits for $(1-\alpha) = 0$, while offering two contracts is strictly superior for $(1-\alpha) > 0$.

From Lemma 2, we know that, if LCS is not IE, the only candidate for an equilibrium offer by the bad firm (outside of region QM(a)) is an overbidding contract menu, which avoids the bad firm's profitable deviations discussed after the Lemma. This contract menu needs to fulfill four requirements in order to be part of a pure-strategy equilibrium. The first two requirements jointly determine the *lower* bound for the utility the bad firm needs to offer to the low type, while the next two requirements define the *upper* bound. Equilibrium existence then requires that the upper bound weakly exceeds the lower bound.[18]

We proceed with an explanation of the bounds, a formal derivation and closed-form representation can be found in Appendix A.3.

Requirements for the lower bound:

- *Binding (PCL)*: As the bad firm has a profitable deviation if *(PCL)* is non-binding, there is no equilibrium where the good firm's best response is in region QC(a). All other parameters given, *(PCL)* in the good firm's best response is binding if the utility the bad firm offers to the low type, U_L^B, weakly exceeds a critical threshold, which we denote by $\widehat{U}_{L,PCL}^B$. The utility offer to the low type then also defines all other parts in the bad firm's contract menu, and via $\Delta\widehat{U}_L^B$ also the contract menu in the good firm's best response. $U_L^B \geq \widehat{U}_{L,PCL}^B$ is required to prevent the bad firm from profitably attracting the *high type only*, which is possible if *(PCL)* is non-binding in the good firm's best response.

- *No profitable cross-subsidy deviation*: Even when the bad firm offers an overbidding contract menu and *(PCL)* is binding in the good firm's best response, the bad firm may still have a profitable deviation if there exists another Pareto-improving contract menu that reduces the upwards distortion in the high type's piece rate by offering even more to the low type. If the efficiency gain in the high type's contract exceeds the losses from the initially considered overbidding contract menu,

[18]Note that the mutual best responses of the bad firm and the good firm are uniquely determined by the lower bound $\widehat{U}_L^B$.

the bad firm can profitably attract *both types*; it would then overcompensate the losses with the low type by the gains from the high type. Therefore, the bad firm needs to overbid to a degree where no cross-subsidy contract menu can yield positive profits. Preventing such a profitable deviation leads to a second lower bound for the utility the bad firm needs to offer to the low type, which we denote by $\widehat{U}^B_{L,CS}$.[19] As the cross subsidy is always offered to the low type, this bound also ensures that the low type gets at least her output.

The overall lower bound $\underline{U}^B_L$ is thus given by $\underline{U}^B_L = \max\left(\widehat{U}^B_{L,PCL}, \widehat{U}^B_{L,CS}\right)$ provided the good firm's best response to $\widehat{U}^B_{L,CS}$ lies outside of Region QM(a); otherwise, $\underline{U}^B_L$ can be lowered further as long as the non-binding *(PCH)* prevents the bad firm from profitably attracting the high type.

Requirements for the upper bound:

- *Participation by the good firm*: The lower bound $\underline{U}^B_L$ just derived implies that the bad firm may need to offer the low type far more than her expected output. The good firm, however, will match the bad firm's contract offer only if it earns non-negative profits with the low type. Otherwise, it offers a contract to the high type with a piece rate sufficiently large to prevent the low type from imitating, and leaves the low type to the bad firm (which, of course, cannot be part of an equilibrium as the bad firm would then face losses).[20] This leads to a first upper bound that we denote by $\widehat{U}^B_{L,\max}$. Due to the good firm's higher productivity, this upper bound is no concern when the bad firm offers both agent types their output, but this may be different for the bad firm's lowest degree of overbidding required to meet the lower bound, i.e., for $U^B_L = \underline{U}^B_L$.

- A second upper bound for the bad firm's overbidding arises from the bad firm's incentive structure itself. Suppose the bad firm offers a high utility U^B_L to the low type, so that the low type has no imitation incentive even when the piece rate for the high type is second-best. As the only benefit of increasing U^B_L is to reduce the upwards distortion in the high type's piece rate, increasing it beyond the point where the bad firm's *(ICCL)* binds even when the high type's piece rate is second-best is weakly dominated. We denote this upper bound by $\widehat{U}^B_{L,noimi}$.

The overall upper bound $\overline{U}^B_L$ is thus given by $\overline{U}^B_L = \min\left(\widehat{U}^B_{L,\max}, \widehat{U}^B_{L,\text{no imi.}}\right)$.

The following Proposition summarizes these insights.

Proposition 5. *(i) There is a pure-strategy equilibrium if and only if* $\underline{U}^B_L \leq \overline{U}^B_L$. *(ii) Any contract menu derived from the mutual best responses with* $U^B_L \in [\underline{U}^B_L, \overline{U}^B_L]$ *constitutes a pure-strategy equilibrium.*

[19] CS refers to "cross subsidy".

[20] Recall from Lemma 3 that, in equilibrium, the good firm employs both types.

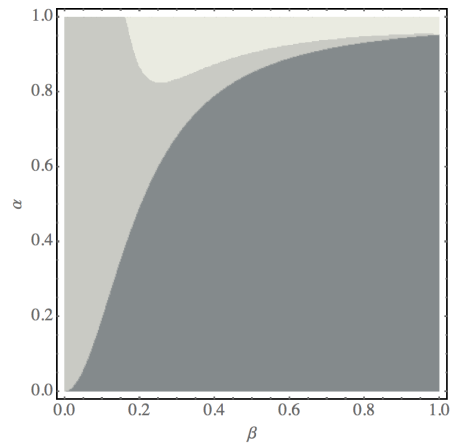

Figure 2: Existence of equilibria. In the dark grey area, the LCS is IE; equilibria exist, but the LCS is not IE in the medium grey area; no equilibria exist in the white area. Parameters are $\theta_H = 2$, $\theta_L = 0.15$, $\rho = 1$, $\sigma = 0.2$.

Based on Proposition 5, we can now deal with the question of the impact of the competitive pressure β on the existence of overbidding equilibria. To answer this question, first recall that, for any α given, the condition for IE of LCS is less restrictive if β is large, so that existence of an equilibrium without overbidding is less problematic if β is large. Such a clear-cut result, however, cannot be derived for overbidding equilibria.

The dark grey area in Figure 2 shows all combinations of α and β for which LCS in the bad firm is IE. For each α-β-pair in this area, there is an equilibrium in which the bad firm offers both agent types their expected output. Recall from Proposition 2 that IE of LCS holds when α is sufficiently small and β sufficiently large. The figure shows that, for all other parameters given, the critical α_{LCS} increases in the bad firm's productivity β. In the medium grey area, LCS in the bad firm is not IE, but overbidding equilibria exist. No equilibrium exists in the white area. The figure illustrates that it may well be the case that, for all other parameters kept constant, an overbidding equilibrium exists for low and for high values of β, but not for values of β in-between.

The intuition for the potential non-monotonicity of equilibrium existence is as follows. Suppose first that β is very low, so that an equilibrium exists in the left part of the medium grey area in the figure. Recall that, in a potentially profitable cross-subsidy strategy, the bad firm offers the low type more than her expected output and the high type less. In doing so, however, it will never offer more to the low type than the high type's expected output. For low β, the high type's expected output in the bad firm is lower than the low type's expected output in the good firm. Therefore, the degree of overbidding required for eliminating potentially profitable cross-subsidy strategies of the bad firm does not violate the requirement that the good firm employs both types. It is therefore perfectly intuitive that overbidding equilibria exist for low levels of β, but may fail to exist when β increases.

However, there is a countervailing effect that may make the condition for equilibrium existence less restrictive as β increases. To see this, recall from our analysis of the impact of risk aversion on IE of LCS that the second-best optimal piece rate, relative to the fixed wage, increases in β. Similar to making the condition for IE of LCS less restrictive, a high β decreases ceteris paribus the bad firm's incentive for large cross subsidies, and a lower overbidding is hence required to eliminate the existence of a profitable cross-subsidy deviation. The non-monotonicity of equilibria existence with respect to β can thus be attributed to the fact that the impact of risk aversion is non-linear.

Finally, we turn to the characterization of overbidding equilibria; with a particular focus on the impact of β on welfare. Observe from Proposition 5 that multiple equilibria exist whenever there exist values for U_L^B that are strictly above the lower and, at the same time, strictly below the upper bound. To analyze the impact of β, it is then important to be consistent in equilibrium selection. For this, we always consider the equilibrium with the lowest degree of overbidding, i.e. the equilibrium where $U_L^B = \underline{U}_L^B$. With this criterion we select the unique perfect equilibrium in pure strategies (as defined in Simon and Stinchcombe (1995)) whenever it exists (which is the case if and only if LCS is IE in the bad firm) and select the pure strategy equilibrium that is "closest" in terms of overbidding if no such perfect equilibrium in pure strategies exists. In Appendix A.3, we show that choosing $\overline{U}_L^B$ instead does not alter the results on the impact of β.

We extend Proposition 4 to include equilibria with overbidding.

Proposition 6. *Suppose that equilibria with $\underline{U}_L^B$ exist in a neighbourhood of β, and assume that* (PCH) *is binding. Then, $\dfrac{\partial \Delta \widehat{U}^B}{\partial \beta} \geq 0$. Moreover, if equilibria with $\underline{U}_L^B$ exist for β_0, β_1 with $\beta_0 < \beta_1$, then $\Delta \widehat{U}^B(\beta_0) \leq \Delta \widehat{U}^B(\beta_1)$.*

Proposition 6 shows that the insights for the impact of competitive pressure on the equilibrium configuration carries over from the case where LCS is interim efficient to the case with overbidding equilibria.

5 Including horizontal differentiation

Vertical differentiation as modelled in the previous sections simplifies the analysis of imperfectly competitive labour market competition because only the good firm employs agents in any pure strategy equilibrium. In this section, we demonstrate that our central findings do not depend on this simplification. To this end, we extend the model to include also a horizontal differentiation (see Bénabou and Tirole (2016)) such that the good and the bad firm hire a positive mass of both types of agents in equilibrium whenever vertical differentiation is not too pronounced. Obviously, a firm hiring a positive mass of both types of agents never overbids as it would otherwise generate a loss and exiting the market would be a profitable deviation. Hence, overbidding equilibria no longer exist. As a consequence, interim efficiency of the LCS allocation for the bad firm is not only sufficient but also necessary for the existence of a pure strategy equilibrium. This equilibrium exhibits the same comparative statics of contracts and welfare with respect to differentiation (now vertical *and* horizontal) as discussed in Proposition 4.

Best Responses The extended model only differs in the assumption that agents are uniformly distributed on the unit interval and the two firms sit on each of its endpoints.[21] Without loss of generality suppose that the good firm (G) is located in $x = 0$ and the bad firm (B) is located in $x = 1$. An agent located at $x \in [0, 1]$ must travel distance x (with cost tx) to firm G and distance $(1 - x)$ (with cost $t(1 - x)$) to firm B. Not considering the vertical differentiation via β, horizontal differentiation (i.e., $t > 0$) resembles a monopsony at $t \to \infty$ and perfect competition at $t \to 0$. We follow Bénabou and Tirole (2016) by assuming that the agent's exogenous outside option $\overline{U}$ (which is normalized to zero as in previous sections) also waits on the endpoints of the interval, i.e., agents must "go and get it" paying tx or $t(1 - x)$. As travelling costs are additive, the agent's effort choice and expected utility (net of travelling costs) from a given contract remains unchanged and she decides to be hired by the firm that offers the larger expected utility with random tie-breaking.

Denote by $x_i^k \in [0, 1]$ the distance between firm $k \in \{G, B\}$ and the agent of type $i \in \{H, L\}$ who is indifferent between the contract offer of the two firms, adopting the convention that $x_i^k = 1$ if all agents of type i prefer the contract offered by firm k and $x_i^k = 0$ if all agents of type i prefer the contract offer by the other firm. We divide firm k's profit Π_i^k from agent type i when offering a contract (F_i^k, w_i^k) into the utility she offers to the agent, U_i^k, and the surplus generated by the agent's effort in response to a bonus w_i^k, denoted by $v_i^k(w_i^k)$, i.e.

$$\Pi_i^k = \frac{1}{2}(1 - w_i^k) w_i^k \beta_k^2 \theta_i^2 - F_i^k \equiv v_i^k(w_i^k) - U_i^k$$

with $v_i^k(w_i^k) = \frac{1}{2}\beta_k^2\theta_i^2 w_i^k(1 - \frac{w_i^k}{2}) - \frac{(w_i^k)^2}{2}\rho\sigma^2$. β is defined as in our basic model and captures the productivity difference between the two firms. Note that $v_i^k(w_i^k)$ is monotone increasing (decreasing) for $w_i^k < (>) w_i^{k,sb} = \frac{\beta_k^2\theta_i^2}{\beta_k^2\theta_i^2 + 2\rho\sigma^2}$, $\frac{dv_i^k(w_i^k)}{dw_i^k} = 0$ for $w_i^k = w_i^{k,sb}$ and $\frac{d^2 v_i^k(w_i^k)}{d(w_i^k)^2} < 0$. With $\widehat{U}_i^{\bar{k}}$ as the utility offered by the competing firm to type i (and multiplying the objective function by $2t$), firm k's optimization program reads

$$\max_{U_H^k, w_H^k, U_L^k, w_L^k} 2t\alpha\, x_H^k\, (v_H^k(w_H^k) - U_H^k) + 2t(1 - \alpha)\, x_L^k\, (v_L^k(w_L^k) - U_L^k), \tag{8}$$

subject to

$$U_L^k \geq U_H^k - \frac{(w_H^k)^2}{4}\beta_k^2(\theta_H^2 - \theta_L^2) \qquad \textit{(ICCL)},$$

$$U_H^k \geq U_L^k + \frac{(w_L^k)^2}{4}\beta_k^2(\theta_H^2 - \theta_L^2) \qquad \textit{(ICCH)}.$$

[21] Following Bénabou and Tirole (2016) we henceforth discuss a continuum of agents. This is equivalent to considering a single agent whose location is uniformly distributed over the unit interval.

For $U_i^k - \widehat{U}_i^{\bar{k}} < -t$ firm $\bar{k}$ is more attractive for type i than firm k regardless of the location and $x_i^k = 0$, for $U_i^k - \widehat{U}_i^{\bar{k}} > t$ firm k is more attractive for type i than firm k regradless of the location and $x_i^k = 1$. If $U_i^k - \widehat{U}_i^{\bar{k}} \in [-t, t]$, $x_i^k = \frac{1}{2} + \frac{U_i^k - \widehat{U}_i^{\bar{k}}}{2t}$. We refer to a best response that satisfies $0 < x_i^k < 1$ as an *interior best response*. As the definition of the critical location x_i^k for an interior best response implies $2tx_i^k = (t + U_i^k - \widehat{U}_i^k)$, the objective function in this case simplifies to

$$\Pi^k = \alpha(t + U_H^k - \widehat{U}_H^k)\Pi_H^k + (1-\alpha)(t + U_L^k - \widehat{U}_L^k)\Pi_L^k.$$

Structure of optimal contracts We will show below that it is optimal for both firms to hire a positive mass of both agent types in any pure strategy equilibrium if, for given travelling costs $t > 0$, vertical differentiation is sufficiently small, i.e., the productivity parameter β for the bad firm is in a sufficiently small open neighborhood of 1. If it is optimal for firm k to hire a positive mass of both types of agents, its best response to contract offers by the competing firm has the same simple structure as in the purely vertically differentiated model.

Lemma 4. *Suppose $0 < x_i^k < 1$ for $i \in \{H, L\}$. Then,*

1. $w_H^{k,*} \geq w_H^{k,sb}$ *and* $w_L^{k,*} \leq w_L^{k,sb}$*;*
2. *If $w_L^{k,*} < w_L^{k,sb}$, then: (i) $w_H^{k,*} = w_H^{k,sb}$; (ii) (ICCH) is binding and (iii) (ICCL) is non-binding.*
3. *If $w_H^{k,*} > w_H^{k,sb}$, then: (i) $w_L^{k,*} = w_L^{k,sb}$; (ii) (ICCL) is binding and (iii) (ICCH) is non-binding.*

By Lemma 4, the same three different types of contract menus as in the purely vertically differentiated case can be optimal interior best responses:[22]

- *Quasi-monopsonic (QM):*
  $w_L^{k,*} < w_L^{k,sb}$, $w_H^{k,*} = w_H^{k,sb}$, $U_H^k - U_L^k = \frac{(w_L^{k,*})^2}{4}\beta_k^2(\theta_H^2 - \theta_L^2)$;
- *Second-best (SB):*
  $w_L^{k,*} = w_L^{k,sb}$, $w_H^{k,*} = w_H^{k,sb}$, $\frac{(w_L^{k,sb})^2}{4}\beta_k^2(\theta_H^2 - \theta_L^2) \leq U_H^k - U_L^k \leq \frac{(w_H^{k,sb})^2}{4}\beta_k^2(\theta_H^2 - \theta_L^2)$;
- *Quasi-competitive (QC):*
  $w_L^{k,*} = w_L^{k,sb}$, $w_H^{k,*} > w_H^{k,sb}$, $U_H^k - U_L^k = \frac{(w_H^{k,*})^2}{4}\beta_k^2(\theta_H^2 - \theta_L^2)$;

In particular, optimal piece rates $w_i^{k,*}$ are either second-best or $(w_i^{k,*})^2$ is proportional to $\Delta U^k = U_H^k - U_L^k$.

[22]By the definition of an interior best response, the participation constraint for the agent of type i located at x_i^k is binding such that no distinction between quasi-competitive and quasi-monopsonic regions with and without a binding participation constraint needs to be made.

Monotone Best Responses Also the monotone comparative statics of best-response contracts extend to the horizontally differentiated model. E.g., the utility difference offered by firm k, $\Delta U^k = U^k_H - U^k_L$, is monotone increasing in the utility difference offered by k's competitor $\Delta\widehat{U}^{\bar{k}} = \widehat{U}^{\bar{k}}_H - \widehat{U}^{\bar{k}}_L$.[23] To show this monotonicity, we use that the firm's objective function in the different regions specified after Lemma 4 is concave whenever the LCS in the bad firm is interim efficient (i.e., $\alpha < \alpha_{LCS}$). As we will show below, IE of LCS in the bad firm is also sufficient for the existence of a pure strategy equilibrium.

Proposition 7. *Suppose $0 < x^k_i < 1$ for $i \in \{H, L\}$ and $\alpha < \alpha_{LCS}$, then*

(i) the best-response utility difference $\Delta U^k = U^k_H - U^k_L$ is monotone increasing in the utility difference offered by the competitor, i.e. $\frac{\partial \Delta U^k}{\partial \Delta \widehat{U}^{\bar{k}}} > 0$;

(ii) there are utility differences offered by the competitor $\Delta\widehat{U}^{\bar{k}}_{QM}$ and $\Delta\widehat{U}^{\bar{k}}_{QC}$ with $\Delta\widehat{U}^{\bar{k}}_{QM} < \Delta\widehat{U}^{\bar{k}}_{QC}$ such that contract offers $(U^k_H, w^k_H; U^k_L, w^k_L)$ are

- quasi-monopsonic *if* $\Delta\widehat{U}^{\bar{k}} < \Delta\widehat{U}^{\bar{k}}_{QM}$.
- second-best *if* $\Delta\widehat{U}^{\bar{k}}_{QM} \leq \Delta\widehat{U}^{\bar{k}} \leq \Delta\widehat{U}^{\bar{k}}_{QC}$
- quasi-competitive *if* $\Delta\widehat{U}^{\bar{k}} > \Delta\widehat{U}^{\bar{k}}_{QC}$

No Exclusion In contrast to the purely vertically differentiated model, also the bad firm hires a positive mass of both types of agents in a pure-strategy equilibrium as long as horizontal differentiation is present (i.e., $t > 0$) and vertical differentiation is not too pronounced (i.e., β is in a sufficiently small open neighborhood of 1).

Lemma 5. *(i) In any pure-strategy equilibrium, the good firm hires a positive mass of both types, i.e. $x_i > 0$ for $i = H, L$.*
(ii) For $t > 0$ there is $\beta_t < 1$ such that the bad firm hires a positive mass of both types of agents if and only if $\beta > \beta_t$.

Interim efficiency of LCS and equilibrium existence For the purely vertically differentiated model, Section 4.1 established interim efficiency (IE) of the least-cost-separating allocation (LCS) as a sufficient condition for the existence of a pure-strategy equilibrium and a necessary condition for a pure-strategy equilibrium without overbidding (unless β is so small that the best response of the good firm lies in region QM(a)). For this finding it was a crucial observation that with IE of LCS, the bad firm had no incentive to offer the low type more than her output in order to reduce the inefficiency for high types. For perfect competition (i.e., $\beta = 1$ and $t = 0$) this implies that no firm has an incentive to offer more than the reservation utility (which is the expected output

[23] The same monotone relation can be shown for all combinations between $(U^k_L, \Delta U^k)$ and $(\widehat{U}^{\bar{k}}_L, \widehat{\Delta} U^{\bar{k}})$ (see the proof of Proposition 7 for further detail).

in this case) to the low type, i.e., the low type does not receive a rent. For $t > 0$, paying no rent to low types means that the low type agent who has to travel a distance of 1 to the firm does not receive more than her reservation utility, i.e., $U_L^k > \widehat{U}_L^k + t$. Bénabou and Tirole (2016) refer to this condition as "no cornering" of low types. As indicated by the following Lemma, IE of LCS indeed implies the absence of cornering incentives.

Lemma 6. *For $t > 0$ there is $\beta_t < 1$ such that for all $\beta_t < \beta < 1$: If the LCS allocation $(F_i^k, w_i^k)_{i=H,L}$ is interim efficient in the bad firm for $t = 0$, i.e.,*

$$\alpha \frac{dv_H^B(w_H^B)}{dw_H^B} + (1-\alpha)\frac{w_H^B}{2}\beta^2(\theta_H^2 - \theta_L^2) \geq 0$$

then $U_i^k \leq \widehat{U}_i^{\bar{k}} + t$ for all $i \in \{H, L\}$ and $k \in \{G, B\}$.

If cornering is not optimal (as can be ensured by IE of LCS in the bad firm for $t = 0$), best responses U_L^k and ΔU^k are continuous and strictly monotone increasing functions of $\widehat{U}_L^{\bar{k}}$ and $\Delta\widehat{U}^{\bar{k}}$ (see Proposition 7). As the strategy space is a lattice, this implies by Milgrom and Roberts (1994, Theorem 3) the existence of a pure strategy equilibrium $C^* = (F_i^{k,*}, w_i^{k,*})$ that resembles a simultaneous solution to the optimization problem (8) for both firms. Now it is easy to see that IE of LCS for $t = 0$ is sufficient but not necessary for the existence of a pure strategy equilibrium: Cornering is profitable for firm k (in response to an offer according to C^* by firm $\bar{k}$ if and only if the marginal gain from reducing the piece-rate distortion in $w_H^{k,*}$ (i.e., $-\alpha x_H^{k,*} {v_H^k}'(w_H^{k,*})$) does not exceed the marginal costs of relaxing the incentive compatibility constraint for low types (i.e., $(1-\alpha)\frac{w_H^{k,*}}{2}\beta_k^2(\theta_H^2 - \theta_L^2)$) (for a formal derivation see the proof of Lemma 6), i.e.,

$$\alpha x_H^{k,*} {v_H^k}'(w_H^{k,*}) + (1-\alpha)\frac{w_H^{k,*}}{2}\beta_k^2(\theta_H^2 - \theta_L^2) \geq 0 \tag{9}$$

Denote the unique α that solves (9) with equality by α_{LCS}^*. As marginal gains from reducing the piece-rate distortion are proportional to the mass of high type agents $x_H^{k,*}$ hired by firm k and $w_H^{k,*}$ is decreasing in t (see Proposition 9), $\alpha_{LCS}^* > \alpha_{LCS}$ for $\beta < 1$ sufficiently large to ensure interior best responses. As stated by the following Proposition, no equilibrium in pure strategies can contain cornering such that the no-cornering condition (i.e., $\alpha < \alpha_{LCS}^*$) is not only sufficient but also necessary for the existence of a pure-strategy equilibrium. We summarize as follows:

Proposition 8. *For $t > 0$ there is $\beta_t < 1$ such that for all $\beta_t < \beta < 1$, there is a pure strategy equilibrium if and only if $\alpha < \alpha_{LCS}^*$.*

The necessity and sufficiency of condition (9) extends the necessity and sufficiency of IE of LCS for the existence of a pure-strategy equilibrium in a model with $\beta = 1$ and $t = 0$ to a model with horizontal and vertical differentiation provided that vertical differentiation is sufficiently weak such that both firms hire both types of agents in equilibrium. As the gains from cornering L types are decreasing in β and increasing in w_H, bad firms have a larger incentive to corner L types and are therefore – as in the purely vertically differentiated model – pivotal for the existence of a pure-strategy equilibrium (recall that unlike the case of purely vertical differentiation, there cannot be any overbidding equilibria if the bad firms hires a positive mass of (both types of) agents). Thus, while pure vertical differentiation allows for equilibria with overbidding, vertical differentiation reduces the support of pure-strategy equilibria where both firms are active. In contrast, horizontal differentiation reduces w_H^* (see Proposition 9) and thereby cornering incentives. The larger the differentiation as expressed through transportation costs t, the larger the support of pure-strategy equilibria determined by $\alpha < \alpha_{LCS}^*$.

Comparative statics As best responses are not only continuous and monotone in the contract offer of the competitor but also in the degree of vertical and horizontal differentiation (β and t), Milgrom and Roberts (1994, Theorem 3) implies that the pure-strategy equilibrium in Proposition 8 also exhibits monotone comparative statics with respect to differentiation.

Proposition 9. *For $t > 0$ and $\alpha < \alpha_{LCS}^*$ there is $\beta_t < 1$ such that for all $\beta_t < \beta < 1$, the pure-strategy equilibrium C^* satisfies $\frac{\partial \Delta U^{k,*}}{\partial t} < 0$ and $\frac{\partial \Delta U^{k,*}}{\partial \beta} > 0$.*

Proposition 9 shows that equilibria in the horizontally differentiated model with both firms hiring both types of agents share all crucial comparative statics with the purely vertically differentiated model. Provided that, in this extended model, vertical differentiation does not prevent the bad firm from hiring at all (which is similar to the purely vertically differentiated case analyzed above), the continuity and strict monotonicity of best responses also implies that equilibria vary in a continuous and monotone fashion in t. Critical threshold differentiations $0 < t_1 < t_2$ exist such that equilibrium contracts are quasi-monopsonistic for $t > t_2$, second-best for $t_1 \leq t \leq t_2$, and quasi-competitive for $t < t_1$.

6 Conclusion

We analyze the impact of imperfect labor market competition on the efficiency of compensation schemes in a model where heterogenous firms compete for risk-averse agents by offering contracts with fixed payments and piece rates. The agents' effort is unobservable and their ability-type is private information. We show that the equilibrium configuration depends crucially on the productivity difference between the two firms, and that three regions need to be distinguished: If the productivity difference between the firms is large (quasi-monopsonistic region), the more productive firm offers a piece rate below the second-best to the low-ability type. Thus, effort incentives for the low type are inefficiently small. As the low type's piece rate increases in the degree of competition in this region, so does social welfare. For intermediate levels of productivity differences, the superior firm offers two second-best piece rates and private information has no impact on equilibrium contracts (second-best region). Finally, if the productivity difference between the two firms is sufficiently small (quasi-competitive region), the more productive firm offers a piece rate above the second-best level to the high type.

Effort incentives are then too high-powered, at the expense of insufficient risk-sharing for the high-ability agent. Social welfare decreases in the degree of competition in this region.

Adding horizontal preferences for firms supports our results and underscores the fact that, with risk-averse agents, a whole range of intermediate competitive levels arises for which second-best contracts are offered to both high- and low-type agents. This result complements the work by Bénabou and Tirole (2016) on horizontal differentiation, where risk neutrality also leads to a hump-shaped relation between efficiency and competition, but where second-best optimal contracts are only offered at one specific level of competition. More generally, our results support current findings that fierce labor market competition induces inefficiently high-powered incentive contracts: Due to severe competition for managerial talent, firms have incentives to offer variable payments to high-ability agents that are above the second-best levels. This result is obtained without introducing limited liability or externalities, that is, even without the factors usually blamed for excessive performance pay, for instance in the financial industry. However, our paper also suggests that labor contracts induce effort incentives that are insufficient in markets where firms have significant market power. In this case, it is the segment of agents with low ability that generates the inefficiency.

A crucial issue in the literature on screening with competing principals is the existence of pure strategy equilibria. In the paper most closely related to ours by Bénabou and Tirole (2016), existence of equilibria is ensured by restricting attention to cases where the least-cost separating allocation is interim efficient. In our main model with only vertical differentiation, we show that the condition for the interim efficiency of the least-cost separating allocation is more restrictive for the bad compared to the good firm. We prove that an equilibrium in which the bad firm would break even when attracting both types exists if and only if the least-cost separating allocation is interim efficient in the bad firm. The maximum proportion of high-type agents up to which this condition holds decreases in the agents' risk aversion, and increases in the bad firm's productivity. However, we also show that vertical differentiation gives rise to a second equilibrium type, in which the bad firm offers the low type more than and the high type exactly her expected output. Such an equilibrium type with "overbidding" by the bad firm emerges due to the fact that, in equilibrium, both agent types are employed by the good firm.

Our modelling of imperfect competition via vertically differentiated firms hence enlarges the set of equilibrium types under adverse selection. In this sense, we complement earlier approaches by Wilson (1977) or Riley (1979) who defined equilibria in a forward-looking manner by allowing firms to anticipate the reaction of competitors to their own contract offers by withdrawing or adding offers. More recent work has built a game-theoretic foundation for these equilibrium concepts and has shown that Wilson-Miyazaki(-Spence) contracts are obtained in equilibrium that may even feature cross-subsidies (Netzer and Scheuer, 2014; Mimra and Wambach, 2011). While we do not resort to these anticipatory equilibrium concepts, our model nevertheless enlarges the parameter space for which equilibria exist, as "overbidding" contracts may constitute equilibria when the least-cost separating allocation is not interim efficient.

Due to the ensuing equilibrium multiplicity, we need to be consistent in equilibrium selection, when analyzing the impact of β on equilibrium properties. Assuming that the bad firm offers the minimum utility to the low type that establishes an equilibrium, we show that higher competition in fact weakly increases the piece rates for both types, which is welfare enhancing in Region QM and detrimental in Region QC. In this respect, competition for agents indeed plays a double-edged role.

Our main economic contribution hence points out how the competitive threat of inferior firms influences the contract design of superior firms, and thereby social welfare. In this sense, our model can be interpreted as capturing the impact of latent rather than actual competition. To consider actual competition, we add horizontal differentiation by assuming that workers have preferences for firms. When workers' preferences are sufficiently strong relative to the productivity difference between firms, then both firms are active in equilibrium. We show that actual competition yields results closely related to those from latent competition with vertically differentiated firms. More precisely, the same three competitive regions need to be distinguished, i.e. intermediate competition remains second-best. Straightforwardly, equilibria with overbidding exist only when the inferior firm is inactive. This proves that the assumption of interim efficiency applied in Bénabou and Tirole (2016) is not only sufficient but necessary for equilibrium existence.

A Formal statements and proofs

A.1 The good firm's best response

Proof of Lemma 1.

1. Deviating from second-best piece rates prevents the respective other agent type from imitating. Both properties, $w_H^* \geq w_H^{sb}$ and $w_L^* \leq w_L^{sb}$, essentially follow from the single-crossing property; that is, increasing the piece rate for the high type, resp. decreasing the piece rate for the low type, increases the utility spread between the low and the high type, which in turn prevents imitation.

Turning to the formal derivation of $w_H^* \geq w_H^{sb}$, consider a set of contracts satisfying all constraints with $w_H^* < w_H^{sb}$. We show that there exists a contract pair satisfying all constraints with piece rate w_H^{sb} that yields higher profit. From the conditions for *(PCH)* and *(ICCH)* we take $U_H(F_H^*, w_H^*) = \max(\widehat{U}_H^B, U_H(F_L^*, w_L^*))$, as any greater utility offered to the high type cannot be optimal. Offering w_H^{sb} and $F_H^{sb} := U_H(F_H^*, w_H^*) - U_H(0, w_H^{sb})$ ensures that *(PCH)* and *(ICCH)* still hold. *(PCL)* does not depend on (F_H^*, w_H^*), so it remains fulfilled. To see that *(ICCL)* still holds, observe that $U_L(F_H^{sb}, w_H^{sb}) = U_H(F_H^*, w_H^*) - \frac{{w_H^{sb}}^2}{4}(\theta_H^2 - \theta_L^2) < U_H(F_H^*, w_H^*) - \frac{{w_H^*}^2}{4}(\theta_H^2 - \theta_L^2) = U_L(F_H^*, w_H^*)$. The principal's profit is increasing in w_H for $w_H < w_H^{sb}$, (see the case with complete information case in Section 2). Since the contract for the low type is unchanged, expected profits are higher than for $w_H^* < w_H^{sb}$.

For $w_L^* \leq w_L^{sb}$, the proof is similar.

2. This is the case where the high type has an imitation incentive if second-best contracts are offered. The standard result holds that the low type's piece rate is distorted to prevent the high type from imitating. The proof is is similar to the proof of part 3 which follows.

3. This it the case where the low type has an incentive to imitate if two second-best contracts were offered. The distortion in the high type's piece rate makes *(ICCL)* binding and prevents the low type from imitating.

Formally, for *(i)*, we show that a set of contracts including $w_L^* = w_L^{sb}$ is optimal. Setting $w_L^* = w_L^{sb}$, choose $F_L^* := \max(\widehat{U}_L^B, U_L(F_H^*, w_H^*)) - \frac{(w_L^*)^2}{4}\left(\theta_L^2 - 2\rho\sigma^2\right)$, which implies $U_L(F_L^*, w_L^*) = \max(\widehat{U}_L^B, U_L(F_H^*, w_H^*))$. Hence, *(PCL)* and *(ICCL)* are fulfilled. Suppose that (F_H^*, w_H^*) are such that *(PCH)* is fulfilled. It remains to show that *(ICCH)* is fulfilled, that is $U_H(F_H^*, w_H^*) \geq U_H(F_L^*, w_L^*)$. Suppose first that *(ICCL)* is binding, $U_L(F_L^*, w_L^*) = U_L(F_H^*, w_H^*)$. Then, using $F_L^* = U_L(F_H^*, w_H^*) - U_L(0, w_L^*)$,

$$\begin{aligned}
&U_H(F_H^*, w_H^*) - U_H(F_L^*, w_L^*) \\
&\qquad = F_H^* + U_H(0, w_H^*) - F_H^* - U_L(0, w_H^*) + U_L(0, w_L^*) - U_H(0, w_L^*) \\
&\qquad = U_H(0, w_H^*) - U_L(0, w_H^*) - (U_H(0, w_L^*) - U_L(0, w_L^*)) \\
&\qquad = \frac{(w_H^{*2} - w_L^{*2})(\theta_H^2 - \theta_L^2)}{4},
\end{aligned}$$

and *(ICCH)* holds since $w_H^* \geq w_H^{sb} \geq w_L^{sb} \geq w_L^*$, as the second-best piece rate increases in θ_i. Now suppose that *(PCL)* is binding instead of *(ICCL)*, that is, $U_L(F_L^*, w_L^*) = \widehat{U}_L^B$, so that $F_L^* = \widehat{U}_L^B - U_L(0, w_L^*)$. Then, together with $U_H(F_H^*, w_H^*) \geq \widehat{U}_H^B$, we have

$$\begin{aligned} U_H(F_H^*, w_H^*) - U_H(F_L^*, w_L^*) &\geq \widehat{U}_H^B - \widehat{U}_L^B + U_L(0, w_L^*) - U_H(0, w_L^*) \\ &= \widehat{U}_H^B - \widehat{U}_L^B - \frac{w_L^{*2}}{4}(\theta_H^2 - \theta_L^2), \end{aligned}$$

which is strictly positive if and only if $\Delta\widehat{U}_B \geq \frac{w_L^{*2}}{4}(\theta_H^2 - \theta_L^2)$. By definition of the case considered, we have $w_H^* > w_H^{sb}$, since *(ICCL)* is violated if the high type is offered w_H^{sb} and F_H^{sb} such that $U_H(F_H^{sb}, w_H^{sb}) = \max(\widehat{U}_H^B, U_H(F_L^*, w_L^*))$. The violated *(ICCL)*-condition can be re-written as $\Delta\widehat{U}_B > \frac{w_H^{sb^2}}{4}(\theta_H^2 - \theta_L^2)$, and the claim then follows since $w_H^{sb} \geq w_L^*$. Finally, recall from the complete information case that this choice of (F_L^*, w_L^*) maximizes the principal's profit from the low type.

For *(ii)*, we show first that *(PCH)* is binding. Assume a contract $(F_H, w_H), (F_L, w_L)$ where *(PCH)* is non-binding and for which *(ICCH)* and *(PCH)* are fulfilled. We show that this contract is not optimal. Choose $\overline{w}_H = w_H$ and $\overline{w}_L = w_L$ and set

$$\overline{F}_H = \widehat{U}_H^B - \frac{\overline{w}_H^2}{4}(\theta_H^2 - 2\rho\sigma^2), \tag{10}$$

$$\overline{F}_L = \max(U_L(\overline{F}_H, \overline{w}_H), \widehat{U}_L^B) - \frac{\overline{w}_L^2}{4}(\theta_L^2 - 2\rho\sigma^2). \tag{11}$$

By construction, $(\overline{F}_H, \overline{w}_H), (\overline{F}_L, \overline{w}_L)$ fulfills *(PCH), (PCL)* and *(ICCL)*. For *(ICCH)* observe first that if *(PCL)* is binding, then

$$U_H(\overline{F}_H, \overline{w}_H) - U_H(\overline{F}_L, \overline{w}_L) = \widehat{U}_H^B - \widehat{U}_L^B - \frac{\overline{w}_L^2}{4}(\theta_H^2 - \theta_L^2),$$

which is strictly positive if and only if $\Delta\widehat{U}_B \geq \frac{w_L^2}{4}(\theta_H^2 - \theta_L^2)$. This holds as we consider the case where $w_H^* > w_H^{sb}$; see the proof of (i). Now suppose that *(ICCL)* is binding. Then,

$$U_H(\overline{F}_H, \overline{w}_H) - U_H(\overline{F}_L, \overline{w}_L) = \frac{w_H^2 - w_L^2}{4}(\theta_H^2 - \theta_L^2) \geq 0.$$

The so-constructed contracts yield higher profits, since the only change is that both fixed wages $\overline{F}_H$ and $\overline{F}_L$ are smaller than F_H and F_L, respectively, cf. Equations (10) and (11).

To show that *(ICCL)* is binding, we use that *(PCH)* is binding, and re-write *(ICCL)* as

$$U_H(F_H, w_H) - U_L(F_H, w_H) \geq \widehat{U}_H^B - U_L(F_L, w_L). \tag{12}$$

For the principal it is optimal to choose $w_H \geq w_H^{sb}$ as small as possible. Furthermore, by the single-crossing property, Equation (3), the left-hand side of Equation (12) is increasing in w_H. Hence, it is optimal to choose w_H such that *(ICCL)* is binding.

For *(iii)*, that is *(ICCH)* non-binding, observe that if *(ICCH)* is binding, then together with the binding *(ICCL)* we have

$$U_H(F_H, w_H) - U_H(F_L, w_L) = F_H - F_L + \frac{w_H^2 - w_L^2}{4}(\theta_H^2 - 2\rho\sigma^2) = 0$$
$$U_L(F_H, w_H) - U_L(F_L, w_L) = F_H - F_L + \frac{w_H^2 - w_L^2}{4}(\theta_L^2 - 2\rho\sigma^2) = 0.$$

This implies $\frac{w_H^2 - w_L^2}{4}(\theta_H^2 - \theta_L^2) = 0$ which requires $w_H = w_L$, i.e., a pooling contract. But this cannot be, as w_L is the second-best piece rate and $w_H \geq w_H^{sb} > w_L$, since the second-best optimal piece rate is increasing in θ_i (cf. Section 2). □

Proof of Proposition 1. If the good firm offers two second-best piece rates and holds both agents on their exit options, the low type has an imitation incentive, whenever

$$U_L(F_H, w_H) > U_L(F_L, w_L) = \widehat{U}_L^B,$$

which can be rewritten as

$$\widehat{U}_H^B - \widehat{U}_L^B > \frac{\theta_H^4(\theta_H^2 - \theta_L^2)}{4(\theta_H^2 + 2\rho\sigma^2)^2} =: \Delta\widehat{U}_{QC}^B,$$

i.e., when the difference in the utilities the bad firm offers to the high and low types is sufficiently large. In turn, the high type has an imitation incentive, whenever

$$U_H(F_L, w_L) > U_H(F_H, w_H) = \widehat{U}_H^B,$$

which can be rewritten as

$$\widehat{U}_H^B - \widehat{U}_L^B < \frac{\theta_L^4(\theta_H^2 - \theta_L^2)}{4(\theta_L^2 + 2\rho\sigma^2)^2} := \Delta\widehat{U}_{QM}^B,$$

i.e., when the utility difference, $\widehat{U}_H^B - \widehat{U}_L^B$, offered by the bad firm is sufficiently small. It is easily verified that $\Delta\widehat{U}_{QC}^B \geq \Delta\widehat{U}_{QM}^B$ (with equality only if $\rho\sigma^2 = 0$), so that no agent has an imitation incentive if $\Delta\widehat{U}^B \in [\Delta\widehat{U}_{QM}^B, \Delta\widehat{U}_{QC}^B]$. In this case it is of course optimal for the good firm to offer second-best piece rates to each agent. This fixes region SB.

Next, we consider the quasi-competitive region QC, in which the low type has an imitation incentive; this corresponds to Part 3 of Lemma 1. The proof for the quasi-monopsonistic region QM is similar.

We can express the good firm's maximization problem as a function of just one variable, w_H. Since the piece rate for the low type is second best, and together with the binding *(PCH)* and *(ICCL)* conditions, we obtain

$$\begin{aligned} F_H(w_H) &= \widehat{U}_H^B - U_H(0, w_H) \\ F_L(w_H) &= U_L(F_H(w_H), w_H) - U_L(0, w_L^{sb}) \\ &= \widehat{U}_H^B - U_H(0, w_H) + U_L(0, w_H) - U_L(0, w_L^{sb}). \end{aligned}$$

The good firm hence solves

$$\max_{w_H} \Pi(w_H) = \alpha\left(\frac{1}{2}(1-w_H)w_H\theta_H^2\right)$$
$$+ (1-\alpha)\left(\frac{1}{2}(1-w_L^{sb})w_L^{sb}\theta_L^2 - U_L(0,w_H) + U_L(0,w_L^{sb})\right) - (\widehat{U}_H^B - U_H(0,w_H)), \quad (13)$$

subject to *(PCL)*, which we write as $\Delta\widehat{U}^B = \widehat{U}_H^B - \widehat{U}_L^B \geq U_H(0,w_H) - U_L(0,w_H)$.

Observe first that $w^* := w_H^*$ is non-decreasing in $\Delta\widehat{U}_B$ as higher $\Delta\widehat{U}_B$ relaxes *(PCL)*, and as $\frac{\partial}{\partial w}[U_H(0,w) - U_L(0,w)] = \frac{w}{2}(\theta_H^2 - \theta_L^2) > 0$. This implies that w^* is strictly increasing in $\Delta\widehat{U}^B$ if *(PCL)* is binding (Region QC(b)).

The partial derivatives are

$$\frac{\partial}{\partial w}\Pi(w) = \frac{1}{2}\left(\alpha\theta_H^2 + w\left((1-2\alpha)\theta_H^2 - (1-\alpha)\theta_L^2 - 2\alpha\rho\sigma^2\right)\right)$$
$$\frac{\partial^2}{\partial w^2}\Pi(w) = \frac{1}{2}\left((1-2\alpha)\theta_H^2 - (1-\alpha)\theta_L^2 - 2\alpha\rho\sigma^2\right).$$

Let w^* solve the FOC $\frac{\partial}{\partial w}\Pi(w^*) = 0$. If $\Delta\widehat{U}^B > U_H(0,w^*) - U_L(0,w^*)$ and $\frac{\partial^2}{\partial w^2}\Pi(w^*) < 0$ (which are the conditions for Region QC(a)), then, because of the binding *(PCH)*, *(PCL)* is fulfilled but non-binding, and $\Pi(w^*)$ is therefore the greatest profit the good firm can derive. Note that $w^* > 0$ if and only if $\frac{\partial^2}{\partial w^2}\Pi(w^*) < 0$. In this case the maximum is global and w^* is constant whenever $\Delta\widehat{U}_B > \frac{\alpha^2\theta_H^4\left(\theta_H^2 - \theta_L^2\right)}{4\left((2\alpha-1)\theta_H^2 + (1-\alpha)\theta_L^2 + 2\alpha\rho\sigma^2\right)^2}$, where the right-hand side corresponds to the threshold where both w^* satisfies the FOC and *(PCL)* is binding.

On the other hand, *(PCL)* is binding whenever $\Delta\widehat{U}_B \leq \frac{\alpha^2\theta_H^4\left(\theta_H^2 - \theta_L^2\right)}{4\left((2\alpha-1)\theta_H^2 + (1-\alpha)\theta_L^2 + 2\alpha\rho\sigma^2\right)^2}$. In this case w^* just solves the binding *(PCL)*.

If $\frac{\partial^2}{\partial w^2}\Pi(w^*) > 0$, then the piece rate that solves the FOC is negative, so that $\Pi(w)$ is strictly increasing on $[0,1]$ and constrained only by the (then binding) *(PCL)* condition, so that this case corresponds to Region QC(b).

Finally, to prove that in Region QC social welfare is decreasing in $\Delta\widehat{U}^B$, observe first that social welfare is given by

$$W(w_H) = \Pi(F_H, w_H, F_L, w_L) + \alpha\, U_H(F_H, w_H) + (1-\alpha)U_L(F_L, w_L)$$
$$= \alpha\left(\frac{1}{2}w_H\theta_H^2 - \frac{1}{4}w_H^2\theta_H^2 - \frac{1}{2}w_H^2\rho\sigma^2\right) + (1-\alpha)\left(\frac{1}{2}w_L\theta_L^2 - \frac{1}{4}w_L^2\theta_L^2 - \frac{1}{2}w_L^2\rho\sigma^2\right),$$

where $w_L = w_L^{sb}$ is constant in Region QC. We thus need to consider the first two derivaties with respect to w_H, which are given by

$$\frac{\partial}{\partial w_H} W(w_H) = \alpha \left(\frac{1}{2}\theta_H^2 - \frac{1}{2} w_H \theta_H^2 - w_H \rho \sigma^2 \right)$$
$$\frac{\partial^2}{\partial w_H^2} W(w_H) = -\alpha \left(\frac{\theta_H^2}{2} + \rho \sigma^2 \right)$$

and $W(w_H)$ is greatest if $w_H = \dfrac{\theta_H^2}{\theta_H^2 + 2\rho\sigma^2} = w_H^{sb}$. Since $w_H^* > w_H^{sb}$ in Region QC, social welfare is decreasing. □

A.2 The least-cost separating allocation is interim efficient

Proof of Proposition 2.

(i). We first show that the low type has an imitation incentive when both types are offered their output and second-best piece rates. In this case, the fixed rates offered by the bad firm are given by $F_i^B(w) = (1-w)\frac{1}{2} w \beta^2 \theta_i^2$, $i \in \{H, L\}$, and the expected utilities for the low type, depending on the contract she choses is given as

$$U_L^B(w_H^{B,sb}) = \frac{1}{2}(1 - w_H^{B,sb}) w_H^{B,sb} \beta^2 \theta_H^2 + \frac{w_H^{B,sb^2}}{4}(\beta^2\theta_L^2 - 2\rho\sigma^2) = \frac{\beta^4 \theta_H^4 (\beta^2\theta_L^2 + 2\rho\sigma^2)}{4(\beta^2\theta_H^2 + 2\rho\sigma^2)^2}$$
$$U_L^B(w_L^{B,sb}) = \frac{1}{2}(1 - w_L^{B,sb}) w_L^{B,sb} \beta^2 \theta_L^2 + \frac{w_L^{B,sb^2}}{4}(\beta^2\theta_L^2 - 2\rho\sigma^2) = \frac{\beta^4 \theta_L^4}{4(\beta^2\theta_L^2 + 2\rho\sigma^2)}.$$

This gives

$$\frac{U_L^B(w_H^{B,sb})}{U_L^B(w_L^{B,sb})} = \frac{(\theta_H^2(\beta^2\theta_L^2 + 2\rho\sigma^2))^2}{(\theta_L^2(\beta^2\theta_H^2 + 2\rho\sigma)^2)^2} = \frac{(\beta^2\theta_L^2\theta_H^2 + 2\rho\sigma^2\theta_H^2)^2}{(\beta^2\theta_L^2\theta_H^2 + 2\rho\sigma^2\theta_L^2)^2} \geq 1,$$

and the claim that the low type has an imitation incentive follows.

Next, we show that there exists $\alpha_{LCS} \in (0,1)$ such that the LCS is interim efficient if and only if $\alpha \leq \alpha_{LCS}$. As the low type has an imitation incentive when both types are offered their expected output, the bad firm offers a pair of quasi-competitive contracts with $w_L^{B,*} = \dfrac{\beta^2\theta_L^2}{\beta^2\theta_L^2 + 2\rho\sigma^2}$ and with the low type's incentive compatibility constraint *(ICCLB)* binding, so that

$$F_H^B(F_L^B, w_H^B) = U_L^B(F_L^B, w_L^{B,*}) - \frac{{w_H^B}^2}{4}(\beta^2\theta_L^2 - 2\rho\sigma^2).$$

The break-even condition is

$$\Pi^B(F_L^B, w_H^B) = \alpha \left\{ \frac{1}{2}(1 - w_H^B) w_H^B \beta^2 \theta_H^2 - U_L^B(F_L^B, w_L^{B,*}) + \frac{{w_H^B}^2}{4}(\beta^2\theta_L^2 - 2\rho\sigma^2) \right\}$$
$$+ (1-\alpha) \left\{ \frac{1}{2} w_L^{B,*}(1 - w_L^{B,*}) \beta^2 \theta_L^2 - F_L^B \right\} = 0.$$

With the Implicit Function Theorem we obtain

$$\frac{\partial w_H^B(F_L^B)}{\partial F_L^B} = -\frac{\frac{\partial}{\partial F_L^B}\Pi^B(F_L^B, w_H^B)}{\frac{\partial}{\partial w_H^B}\Pi^B(F_L^B, w_H^B)} = -\frac{2}{\alpha(\beta^2(\theta_H^2(2w_H^B - 1) - \theta_L^2 w_H^B) + 2\rho\sigma^2 w_H^B)}.$$

When increasing the low type's utility by increasing F_L^B, the high type's utility changes according to

$$\begin{aligned}\frac{\partial}{\partial F_L^B} U_H^B(F_H^B(F_L^B), w_H^B(F_L^B)) &= \frac{\partial}{\partial F_L^B}\left\{U_L^B(F_L^B, w_L^B) + \frac{\beta^2 w_H^B(F_L^B)^2}{4}(\theta_H^2 - \theta_L^2)\right\} \\ &= 1 + \frac{w_H^B(F_L^B)\, w_H^{B\prime}(F_L^B)\beta^2}{2}(\theta_H^2 - \theta_L^2) \\ &= 1 - \frac{w_H^B\beta^2}{\alpha(\beta^2(\theta_H^2(2w_H^B - 1) - \theta_L^2 w_H^B) + 2\rho\sigma^2 w_H^B)}(\theta_H^2 - \theta_L^2).\end{aligned}$$

This expression is positive if and only if

$$\alpha \geq \frac{w_H^B\beta^2(\theta_H^2 - \theta_L^2)}{w_H^B\beta^2(\theta_H^2 - \theta_L^2) + \beta^2\theta_H^2(w_H^B - 1) + 2\rho\sigma^2 w_H^B} = \frac{w_H^B\beta^2(\theta_H^2 - \theta_L^2)}{w_H^B\beta^2(\theta_H^2 - \theta_L^2) - 2\frac{dv_H^B(w_H^B)}{dw_H^B}}. \quad (14)$$

In particular, if w_H^B refers to a contract pair where each type receives exactly her output,[24] then (14) implies that the high type's certainty equivalent can be increased by offering her less than her output, provided that the right-hand side of (14) is smaller than 1. This is the case if and only if

$$\beta^2\theta_H^2(w_H^B - 1) + 2\rho\sigma^2 w_H^B > 0.$$

This is equivalent to $w_H^B > \dfrac{\beta^2\theta_H^2}{\beta^2\theta_H^2 + 2\sigma\rho^2} = w_H^{B,sb}$ which holds for all quasi-competitive contract menus.

Summing up, there exists $\alpha < 1$ such that both the high type's and the low type's utility increases when deviating from a contract where each type receives her output, while maintaining the break-even condition.

(ii) and **(iii)**. Note further that (14), with the appropriate w_H^B where each type receives exactly her output, implies that $\dfrac{\partial}{\partial\beta}\alpha_{LCS} \geq 0$ and $\dfrac{\partial}{\partial\rho}\alpha_{LCS} \leq 0$. □

[24] To derive an explicit expression for w_H^B, note first that when each type receives her output, then

$$\begin{aligned}F_L^B &= \frac{1}{2}(1 - w_L^B)w_L^B\beta^2\theta_L^2 \\ F_H^B &= \frac{1}{2}(1 - w_H^B)w_H^B\beta^2\theta_H^2.\end{aligned}$$

From the binding *(ICCLB)*, we can solve for w_H^B:

$$w_H^B = \frac{\beta^2\theta_H^2 + \sqrt{\beta^4\theta_H^4 - 4U_L^B(F_L^B, w_L^B)(\beta^2(2\theta_H^2 - \theta_L^2) + 2\rho\sigma^2)}}{\beta^2(2\theta_H^2 - \theta_L^2) + 2\rho\sigma^2}$$

Proof of Proposition 3. Suppose the bad firm offers LCS contracts (i.e., both types receive their expected output in the bad firm) and the good firm best responds with a contract menu as specified in Proposition 1. The second assumption is without loss of generality as we will show in Lemma 3 that the good firm hires both types of agents in any pure strategy equilibrium.

Case 1: (PCH) non-binding.

Suppose the good firm offers a contract to the high type such that (PCH) is non-binding. By Proposition 1, this implies that (PCL) is binding. Then, the bad firm has no profitable deviation since LCS is IE and hiring any type of agent requires to offer more than their output. In particular, offering more to the high type without attracting her, does not allow for a deviation that profitably attracts the low type. Hence, if LCS is IE and the good firm best responds with (PCH) not binding (i.e., a quasi-monopsonistic contract), there exists a pure strategy equilibrium.

Case 2: (PCH) binding.

Let the good firm offer a contract to the high type such that (PCH) is binding. Then, (PCL) for the good firm's contract offer has to be binding as well. To see this suppose that (PCL) is not binding. Then, the bad firm could increase the utility offered to the low type without attracting her such that the inefficiently high piece rate for the high type could be reduced. This generates more surplus and the high type could be profitably attracted by the bad firm which resembles a profitable deviation. But if (PCL) and (PCH) are binding, the bad firm has no profitable deviation since LCS is IE and hiring any type of agent requires to offer more than their output. Hence, if LCS is IE and the good firm best responds with (PCH) and (PCL) binding, there exists a pure strategy equilibrium. It remains to show that the best response according to Proposition 1 indeed yields a binding (PCL) – i.e., the best response is not in region QC(a). To see this suppose hypothetically that the *good* firm offers both types of agents their output. If LCS is IE in the bad firm, then this particular allocation is also IE because α_{LCS} is increasing in β (see Proposition 2(ii)). Hence, the good firm does not benefit from increasing the low type's rent (and reducing the high type's piece rate distortion) when offering the output to both types of agents. So *if* offering both types their output is a best response for the good firm, (PCL) is binding and the offer is in region QC(b) of Proposition 1. When offering both types of agents their output, $\Delta U^B = U_H^B - U_L^B$ is non-decreasing in β (see Proposition 4). So $\Delta\widehat{U}^B$ for $\beta < 1$ is bounded from above by $\Delta\widehat{U}^B$ for $\beta = 1$. By Proposition 1, $\Delta\widehat{U}^B$ is larger in region QC(a) than in region QC(b). We already saw that the best response against $\Delta\widehat{U}^B$ for $\beta = 1$ (which is offering both types their output) is in region QC(b). Therefore best responses against $\Delta\widehat{U}^B$ for $\beta < 1$ can never be in region QC(a) and (PCL) is indeed binding. $\square$

Proof of Proposition 4. As the bad firm offers a quasi-competitive contract, the incentive compatibility constraint of the low type in the bad firm is binding, i.e., $U_L^B = U_H^B - \frac{(w_H^B)^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$. Hence, $\Delta\widehat{U}^B$ is increasing in β for a given w_H^B and increasing in

w_H^B. The optimal choice of w_H^B equilibrates the marginal loss due to additional rents for low types (i.e., $(1-\alpha)\frac{(w_H^B)^2}{4}\beta^2(\theta_H^2-\theta_L^2)))$ and the marginal gain from reducing piece-rate distortions for high types (i.e., $-\alpha\frac{dv_H^B(w_H^B)}{dw_H^B}$). Taking the first order condition for w_H^B (i.e., $\alpha v_H^{B\prime}(w_H^B)+(1-\alpha)\frac{w_H}{2}(\theta_H^2-\theta_L^2)=0$) as an implicit function, we get $\frac{dw_H^B}{d\beta}>0$ if $\alpha<\alpha_{LCS}$. It follows that $\frac{d\Delta\widehat{U}^B}{d\beta}>0$.

The second statement follows in a similar way. $\square$

A.3 The least-cost separating allocation is *not* interim efficient

Proof of Lemma 2.

(i). We first show that the bad firm offers the high type exactly her expected output. This consists of two parts: First, if she is offered less and *(PCH)* is binding, then the bad firm has a profitable deviation. Second, offering more is weakly dominated as the high type has no imitation incentive in the bad firm.

Suppose the bad firm offers contracts $(F_H^{B,*},w_H^{B,*}),(F_L^{B,*},w_L^{B,*})$ where the high type is offered less than her output. Then, since *(PCH)* is binding, the bad firm has the following profitable deviation: It can increase its offer to the high type and profitably attract her. Of course, the low type must not be attracted, and therefore the profitable deviation entails a greater distortion in the contract designed for the high type. Formally, choose $(\overline{F}_H^B,\overline{w}_H^B)$ with $\overline{w}_H^B>w_H^{B,*}$ and

$$\overline{F}_H^B(\overline{w}_H^B)=U_L^B(F_H^{B,*},w_H^{B,*})-U_L^B(0,\overline{w}_H^B)=F_H^{B,*}+\frac{{w_H^{B,*}}^2-{\overline{w}_H^B}^2}{4}(\beta^2\theta_L^2-2\rho\sigma^2).$$

By construction, $U_L^B(\overline{F}_H^B,\overline{w}_H^B)=U_L^B(F_H^{B,*},w_H^{B,*})$, so *(ICCLB)* is satisfied. Furthermore,

$$\begin{aligned}
&U_H^B(\overline{F}_H^B,\overline{w}_H^B)-U_H^B(F_H^{B,*},w_H^{B,*})\\
&\qquad=U_L^B(F_H^{B,*},w_H^{B,*})-U_L^B(0,\overline{w}_H^B)+U_H^B(0,\overline{w}_H^B)-U_H^B(F_H^{B,*},w_H^{B,*})\\
&\qquad=\frac{{\overline{w}_H^B}^2-{w_H^{B,*}}^2}{4}(\beta^2\theta_H^2-\beta^2\theta_L^2)>0,
\end{aligned}$$

which implies that *(ICCHB)* is satisfied, and that the bad firm attracts the high type because *(PCH)* was binding.

It remains to show that there exists $\overline{w}_H^B$ such that the expected profit from the high type, $\Pi_H^B(\overline{F}_H^B,\overline{w}_H^B)$, is positive. But this follows directly by observing that

$$\Pi_H^B(\overline{F}_H^B(\overline{w}_H^B),\overline{w}_H^B)=\alpha\left\{\frac{1}{2}(1-\overline{w}_H^B)\overline{w}_H^B\beta^2\theta_H^2-F_H^{B,*}-\frac{{w_H^{B,*}}^2-{\overline{w}_H^B}^2}{4}(\beta^2\theta_L^2-2\rho\sigma^2)\right\}$$

is continuous in $\overline{w}_H^B$, and that $\Pi_H^B(F_H^{B,*}, w_H^{B,*}) > 0$ as the high type is offered less than her output. Thus, a profitable deviation exists when the bad firm offers the high type less than her expected output.

Next, the only reason to offer the high type more than her expected output would be to reduce her imitation incentive. Otherwise, this is weakly dominated. It is hence sufficient to show that the high type has no imitation incentive when the bad firm offers two second-best contracts. It was already shown in the proof of Proposition 3 that it is the low type who has an imitation incentive when two second-best contracts are offered where each type receives her expected output, which implies that the high type has no incentive to imitate. Let us now loosen the assumption that the low type is offered exactly her expected output. Given the previous result, offering the low type less than her expected output cannot result in an imitation incentive. The only reason to offer the low type more than her output is to reduce her imitation incentive, and to profitably attract the high type with a more attractive contract. Of course, such a contract does not exist when the high type's piece rate is second-best, and then, offering the low type more than her output is weakly dominated.

(ii). We show that there exists a profitable deviation that allows the bad firm to attract the high type: when *(PCL)* is non-binding, the bad firm can increase its offer to the low type without attracting her, which allows to reduce the inefficiency in the piece rate for the high type. Suppose contracts (F_H^*, w_H^*), (F_L^*, w_L^*), $(F_H^{B,*}, w_H^{B,*})$, $(F_L^{B,*}, w_L^{B,*})$ with a non-binding *(PCL)*, $U_L(F_L^*, w_L^*) > U_L^B(F_L^{B,*}, w_L^{B,*})$, are offered. Choose $(\overline{F}_L^B, \overline{w}_L^B), (\overline{F}_H^B, \overline{w}_H^B)$ such that

$$\overline{w}_L^B = 0, \tag{15}$$

$$\overline{F}_L^B = U_L(F_L^*, w_L^*) =: \widehat{U}_L^G, \tag{16}$$

$$\overline{F}_H^B = \widehat{U}_L^G - U_L^B(0, \overline{w}_H^B), \tag{17}$$

$$U_H^B(\overline{F}_H^B, \overline{w}_H^B) = U_H(F_H^*, w_H^*). \tag{18}$$

Equations (15) and (16) imply that *(PCL)* is binding. Because of Equation (18), the contract $(\overline{F}_H^B, \overline{w}_H^B)$ does not attract the high type, but once we have verified that this contract pair fulfills the constraints and that the high type receives less than her expected output, the existence of a profitable deviation follows from part (i).

By construction, *(PCL)* and *(ICCLB)* are binding. *(ICCHB)* is fulfilled since

$$U_H^B(\overline{F}_H^B, \overline{w}_H^B) = \widehat{U}_H > \widehat{U}_L = \overline{F}_L^B,$$

where the first equality follows from the binding *(PCH)*, which follows from the initially non-binding *(PCL)*.

To see that the high type receives less than her expected output, we first show that $w_H^{B,*} > \overline{w}_H^B$: From *(ICCLB)* in the initial contract, we have

$$U_H^B(F_H^{B,*}, w_H^{B,*}) \leq U_L^B(F_L^{B,*}, w_L^{B,*}) + \frac{\beta^2 {w_H^{B,*}}^2}{4}(\theta_H^2 - \theta_L^2).$$

Furthermore,

$$U_H^B(F_H^{B,*}, w_H^{B,*}) = U_H^B(\overline{F}_H^B, \overline{w}_H^B) = \widehat{U}_L^G + \frac{\beta^2 \overline{w}_H^{B\,2}}{4}(\theta_H^2 - \theta_L^2),$$

so that

$$\frac{\beta^2}{4}(w_H^{B,*\,2} - \overline{w}_H^{B\,2})(\theta_H^2 - \theta_L^2) \geq \widehat{U}_L^G - U_L^B(F_L^{B,*}, w_L^{B,*}) > 0,$$

which implies that $w_H^{B,*} > \overline{w}_H^B$.

Next, because the contract offer by the bad firm is quasi-competitive, it follows that $\overline{w}_H^B > w_H^{B,sb}$. Together with $w_H^{B,*} > \overline{w}_H^B$, it follows that the high type receives less than her expected output: Since $w_H^{B,*} > \overline{w}_H^B > w_H^{B,sb}$, her certainty equivalent is higher with $\overline{w}_H^B$ compared to $w_H^{B,*}$ if offered her expected output in both cases. And as her certainty equivalent is unchanged by construction via the choice of $\overline{F}_H^B$, the claim follows.

(iii). Given the analysis of part (i) it suffices to observe that, whenever *(PCL)* is binding, offering the low type less than her output and less than the second-best piece rate allows the bad firm to profitably attract the low type. □

Proof of Lemma 3. Note first that, with exogenous reservation levels of utility $\widehat{U}_i$ and without further restrictions, it could of course be profit-maximizing to employ just one agent type. For instance, if the high type's reservation level of utility, $\widehat{U}_H$, were greater than her output with a second-best piece rate, then it would be optimal to employ only the low type. In our model, however, the reservation levels of utility are endogenously derived from the bad firm's offers, and hence bounded from above. We show that, given these upper bounds, the good firm can never increase its profit by hiring just one agent type. We distinguish two cases:

Case 1. (PCL) and (PCH) are both binding.

Suppose first that the good firm hires both types, as assumed in the best response function (6) and that *(PCL)* and *(PCH)* are binding in its best response. Recall that this is the case in Regions QM(b), SB and QC(a). Due to the good firm's productivity advantage, the bad firm can only derive profit from attracting a single type or both types if this is also the case for the good firm. Therefore, the bad firm will not bid more for a type than is profitable for the good firm. When the good firm offers two contracts, then at most one contract is distorted to prevent the other type from imitating. Offering no contract instead of a distorted contract for the respective type foregoes any positive expected profit from this type. All other contracts are second-best, and profits cannot be increased by not offering contracts due to the binding *(PC)s*. Therefore, not placing an offer for any one type cannot be a profitable deviation.

Case 2. (PCH) is non-binding.

The non-trivial case arises in Region QM(a), where *(PCH)* is non-binding, so that the high type receives an information rent if attracted. For this case, we know from textbook models that, whenever the probability of meeting a low type, $1 - \alpha$, is sufficiently small, it may be profitable to hire only the high type in order to save the information rent. We show that this is not the case in our model. The reason is that the high type's information rent depends on α and, in particular, as $1-\alpha$ tends to 0, so does the high type's information rent. As a consequence it turns out that the profits from offering two contracts versus offering only one contract converge as $1-\alpha \to 0$, with offering two contracts being strictly more profitable than offering only one for arbitrary $1-\alpha > 0$.

Formally, in Region QM(a), when offering two contracts we have

$$\begin{aligned}
F_L(w_L) &= \widehat{U}_L^B - U_L(0, w_L) && \text{(binding } \textit{(PCL)}\text{)} \\
F_H(w_L) &= U_H(F_L, w_L) - U_H(0, w_H) && \text{(binding } \textit{(ICCH)}\text{)} \\
w_H &= w_H^{sb}
\end{aligned}$$

Furthermore, w_L solves the FOC of the good firm's profit function, and is given by

$$w_L = \frac{\theta_L^2}{\theta_L^2 + 2\rho\sigma^2 + \alpha/(1-\alpha)(\theta_H^2 - \theta_L^2)}.$$

Let us analyse the difference in offering two contracts with non-binding *(PCH)* versus offering one contract with binding *(PCH)*, which is given by (assuming that $\Delta\widehat{U}^B$ is small enough so that we are in Region QM(a))

$$\Delta = \Pi(F_H(w_L), w_H, F_L(w_L), w_L) - \alpha\left(\frac{1}{2}(1-w_H)w_H\theta_H^2 - (\widehat{U}_H^B - U_H(0, w_H))\right).$$

Simplifying yields

$$\Delta = \frac{(1-\alpha)^2\theta_L^4}{4(\alpha(\theta_H^2 - \theta_L^2) + (1-\alpha)(\theta_L^2 + 2\rho\sigma^2))} + \alpha\widehat{U}_H^B - \widehat{U}_L^B. \tag{19}$$

When $\beta = 0$, then both $\widehat{U}_H^B = 0$ and $\widehat{U}_L^B = 0$, and only the first term (a positive constant) remains. On the other hand, on the boundary between Regions QM(a) and QM(b), where *(PCH)* becomes binding, Δ is just the profit derived from the low type. In an equilibrium where the bad firm hires the low type, this would be strictly positive, as (i) $\beta < 1$ (when $\beta = 1$ we are in Region QC, as the low type has an imitation incentive) and (ii) the bad firm would pay the low type at most her output (otherwise the bad firm would not want to hire the low type).

It remains to analyse whether $\Delta > 0$ for $\beta > 0$ in Region QM(a). Since the explicit expression for $\widehat{U}_H^B$ is too involved, we find a lower bound for Δ. First, observe that the constant term in Δ (the first term on the RHS of Equation (19)) can be re-written as

$$\begin{aligned}
\frac{(1-\alpha)^2\theta_L^4}{4(\alpha(\theta_H^2 - \theta_L^2) + (1-\alpha)(\theta_L^2 + 2\rho\sigma^2))} &= \frac{w_L^2}{4}(\alpha(\theta_H^2 - \theta_L^2) + (1-\alpha)(\theta_L^2 + 2\rho\sigma^2)) \\
&\geq \frac{{w_H^B}^2}{4}\beta^2(\alpha(\theta_H^2 - \theta_L^2) + (1-\alpha)(\theta_L^2 + 2\rho\sigma^2)), \quad (20)
\end{aligned}$$

where the inequality follows since $w_L = \beta w_H^B$ when *(PCH)* is binding and $w_L > \beta w_H^B$ when *(PCH)* is non-binding. [25] Inserting the lower bound for the constant term from Equation (20), we obtain after simplifying

$$\Delta \geq \frac{1}{2}(1-\alpha) w_H^B \left(\beta^2(\theta_H^2(w_H^B - 1) - w_H^B(\theta_L^2 + \rho\sigma^2)) + \rho\sigma^2 w_H^B\right) =: \Delta'.$$

At $\beta = 0$, we have $\Delta' = \frac{1}{2}(1-\alpha)\rho\sigma^2 {w_H^B}^2 > 0$, whereas on the boundary between Regions QM(a) and Region QM(b), we have $\Delta' = \Delta \geq 0$. Furthermore, Δ' as a function of β has at most one zero on $[0, 1]$, so that we can conclude that $\Delta' \geq 0$ in Region QM(a). □

Full specification of equilibrium configuration Before we proceed to the proof of Proposition 5, we first discuss the full specification of the equilibrium configuration. We already know that the bad firm offers both types at least their expected output,[26] cross-subsidizes the low type if the LCS is not interim efficient, and needs to offer the low type a utility such that *(PCL)* is binding in the good firm's best response. These requirements jointly set a *lower bound* on the minimum utility the bad firm offers the low type in equilibrium. At the same time, the good firm's willingness to compete sets an *upper bound*. If the upper bound is below the lower bound, there is no pure-strategy equilibrium, while we have multiple equilibria if it is strictly above.

In order to derive these bounds and the range of equilibria, we need to make the bad firm's best response explicit. In equilibrium, the bad firm's best response satisfies

$$U_L^B(F_L^{B,*}, w_L^{B,*}) = \widehat{U}_L^G = \widehat{U}_L^B \tag{21a}$$

$$F_H^{B,*} = \frac{1}{2}(1 - w_H^{B,*}) w_H^{B,*} \beta^2 \theta_H^2 \tag{21b}$$

$$w_H^{B,*} \text{ such that } U_L^B(F_H^{B,*}, w_H^{B,*}) = \widehat{U}_L^G. \tag{21c}$$

Equation (21a) states that *(PCL)* is binding, $\widehat{U}_L^G = \widehat{U}_L^B$. The expression for $F_H^{B,*}$ in (21b) implies that the high type receives her expected output, and the expression for

[25] When *(PCH)* is binding we have in Region QM, because of the binding *(ICCH)*,

$$\Delta \widehat{U}^B = U_H(F_L, w_L) - U_L(F_L, w_L) = \frac{w_L^2}{4}(\theta_H^2 - \theta_L^2).$$

In the bad firm, we have because of the binding *(ICCLB)*,

$$\Delta \widehat{U}^B = U_H^B(F_H^B, w_H^B) - U_L^B(F_H^B, w_H^B) = \frac{{w_H^B}^2}{4}\beta^2(\theta_H^2 - \theta_L^2),$$

so that in Region QM(b) we have $w_L = w_H^B \beta$. For Equation (20), we have equality on the boundary between Regions QM(a) and QM(b) and strict inequality in Region QM(a).

[26] In the case where *(PCH)* is non-binding (Region QM(a)), such a contract specification is contained in the bad firm's best response set. Since the exact specification of the high type's contract has no effect on the good firm's best response, we take this particular contract specification as given in concrete calculations, but it should be stressed that this has no effect on the good firm's best response, existence of equilibria and impact of competition.

$w_H^{B,*}$ in (21c) says that *(ICCLB)* is binding. Further, the piece rate for the low type is second-best as any other piece rate is weakly dominated.

Summing up, a pure-strategy equilibrium requires that the best responses of the good and the bad firm are simultaneously given by Equation (6) and Equations (21a)–(21c), respectively, and that the good firm's expected profit from each agent is non-negative. We hence need to establish under which conditions the best responses are given by Equation (6) and Equations (21a)–(21c).

For this, we can express an entire equilibrium configuration via one variable, such as $\widehat{U}_L^B$, the utility the low type is offered by the bad firm: Given $\widehat{U}_L^B$, we can determine $\widehat{U}_H^B$ such that the low type does not imitate and such that the high type receives her expected output in the bad firm. The binding *(PCL)* and $\Delta\widehat{U}^B$ then determine the best response by the good firm.

Proof of Proposition 5. We show that, if $\widehat{U}_L^B \in [\underline{U}_L^B, \overline{U}_L^B]$, then the mutual best responses of both firms are given by Equation (6) for the good firm, resp. (21a)–(21c) by the bad firm, and that a profitable deviation exists for at least one firm if $\widehat{U}_L^B \notin [\underline{U}_L^B, \overline{U}_L^B]$.

In the following denote by $\widehat{U}_{L,LCS}^B$ the utility offered to the low type if the bad firms' LCS is IE. In this case, the low type receives her expected output from a contract with a second-best piece rate; formally: $\widehat{U}_{L,LCS}^B = \frac{\left(w_L^{B,sb}\right)^2}{4}\beta^2\theta_L^2$. Note that $\widehat{U}_{L,CS}^B \geq \widehat{U}_{L,LCS}^B$ as a potential cross-subsidy offers the low type more than her expected output.

Consider first the bad firm whose best response is given by (21a)–(21c): Whenever $\widehat{U}_L^G \geq \widehat{U}_{L,LCS}^B$, there is no profitable deviation to attract solely the low type. Thus, the only profitable deviation aims at attracting the high type or both types. We distinguish two cases, depending on whether the bad firm's best response leads to a binding, respectively non-binding, *(PCH)* in the good firm.

Case 1. (PCH) *is binding.*

In this case, the minimum utility to be offered to the high type so that she cannot be profitably attracted regardless of the actions of the good firm is given by $\widehat{U}_{L,CS}^B$. We formalize the minimum certainty equivalent $\widehat{U}_{L,CS}^B$ to be offered to the low type as result of a potential cross-subsidising strategy between the high and low types. Taking into account that in the good firm's best response *(PCH)* is binding, the bad firm may have a profitable deviation by offering the high type less than her output, while offering more to the low type, since increasing the low type's information rent decreases the piece rate offered to the high type, thus reducing the inefficiency.

The highest utility the bad firm can offer to the high type without incurring a loss (in expectation) when attracting her or both types is given by

$$\widehat{U}_{H,CS}^B := \max_{F_H^B, w_H^B, F_L^B, w_L^B} U_H^B(F_H^B, w_H^B) \tag{22}$$

subject to *(ICCLB), (ICCHB)* and

- $\alpha\left(\frac{1}{2}(1-w_H^B)w_H^B\beta^2\theta_H^2 - F_H^B\right)$
$$+ (1-\alpha)\left(\frac{1}{2}(1-w_L^B)w_L^B\beta^2\theta_L^2 - F_L^B\right) = 0 \quad \text{(break-even)},$$
- $U_L(F_L^B, w_L^B) \geq \widehat{U}_{L,LCS}^B$ (output L).

Any offer by the good firm to the high type with $\widehat{U}_H < \widehat{U}_{H,CS}^B$ will be outbid by the bad firm, so that $\widehat{U}_{H,CS}^B$ is the smallest certainty equivalent to be offered to the high type. The explicit expression for $\widehat{U}_{H,CS}^B$ given below is derived as follows: Using that *(ICCLB)* is binding, the break-even condition and $w_L^B = w_L^{B,sb}$, the high type's certainty equivalent is a function of her piece rate w_H^B only. Maximizing Equation (22) via the first-order condition yields

$$\widehat{U}_{H,CS}^B = \frac{\alpha^2\beta^4\theta_H^4}{4(\beta^2((2\alpha-1)\theta_H^2 + (1-\alpha)\theta_L^2) + 2\alpha\rho\sigma^2)} + \frac{(1-\alpha)\beta^4\theta_L^4}{4(\beta^2\theta_L^2 + 2\rho\sigma^2)}. \tag{23}$$

This needs to be translated into the smallest certainty equivalent to be offered to the low type, which is given by

$$\widehat{U}_{L,CS}^B = U_L^B(F_H^B, w_H^B),$$

with $U_H^B(F_H^B, w_H^B) = \widehat{U}_{H,CS}^B$ and $F_H^B = \frac{1}{2}(1-w_H^B)w_H^B\beta^2\theta_H^2$. This offer ensures that the high type is offered $\widehat{U}_{H,CS}^B$ while fulfilling Equations (21a)–(21c). The explicit expression for $\widehat{U}_{L,CS}^B$ is then given by

$$\widehat{U}_{L,CS}^B = \widehat{U}_{H,CS}^B - \frac{\beta^2\left(\beta^2\theta_H^2 + \sqrt{\beta^4\theta_H^4 - 4\widehat{U}_{H,CS}^B(\beta^2\theta_H^2 + 2\rho\sigma^2)}\right)^2}{4(\beta^2\theta_H^2 + 2\rho\sigma^2)^2}(\theta_H^2 - \theta_L^2),$$

if condition (output L) of the optimisation problem (22) is fulfilled. Note that if (output L) is binding, then the bad firm's LCS is IE and $\widehat{U}_{L,CS}^B = \widehat{U}_{L,LCS}^B$.

Recall next from Lemma 2, that a non-binding *(PCL)* entails a profitable deviation for the bad firm. If *(PCL)* is binding for certainty equivalent $\widehat{U}_L^G$, then it is also binding for all greater certainty equivalents. To see this, observe that it follows directly from Equations (21a)–(21c) that $w_L^{B,*} = w_L^{B,sb}$ and $F_L^{B,*} = \widehat{U}_L^G - U_L^B(0, w_L^{B,sb})$. The explicit solution for the high type's piece rate in the bad firm's best response is then given by[27]

$$w_H^{B,*} = \frac{\beta^2\theta_H^2 + \sqrt{\beta^4\theta_H^4 - 4\widehat{U}_L^G(2\beta^2\theta_H^2 - \beta^2\theta_L^2 + 2\rho\sigma^2)}}{2\beta^2\theta_H^2 - \beta^2\theta_L^2 + 2\rho\sigma^2}. \tag{24}$$

[27] Existence of a solution in $\mathbb{R}$ follows from $w_H^{B,*} \geq w_H^{B,sb}$, and since the right-hand side of Equation (24) is decreasing in $\widehat{U}_L^G$ and smaller than $w_H^{B,sb}$ when the expression in the square root is 0.

Hence, (24) implies $\dfrac{\partial w_H^B}{\partial \widehat{U}_L^G} \leq 0$, which implies that $\Delta\widehat{U}^B = \dfrac{{w_H^B}^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ decreases in $\widehat{U}_L^G$. Furthermore, we know from Proposition 1 that *(PCL)* remains binding as U_L^G increases. Hence, the bad firm's best response to $\widehat{U}_L^G > \max(\widehat{U}_{L,CS}^B, \widehat{U}_{L,PCL}^B)$ does not allow her to profitably attract any type.

The explicit expression for $\widehat{U}_{L,PCL}^B$ is determined by the threshold that separates Regions QC(a) and QC(b) in Proposition 1, that is, where

$$\Delta\widehat{U}^B = \frac{\alpha^2\theta_H^4\left(\theta_H^2 - \theta_L^2\right)}{4\left((2\alpha-1)\theta_H^2 + (1-\alpha)\theta_L^2 + 2\alpha\rho\sigma^2\right)^2}.$$

By the binding *(ICCLB)* constraint we have that

$$\Delta\widehat{U}^B = U_H^B(F_H^B, w_H^B) - U_L^B(F_H^B, w_H^B) = \frac{{w_H^B}^2}{4}\beta^2(\theta_H^2 - \theta_L^2),$$

so that

$$w_H^B = \frac{\alpha\theta_H^2}{\beta((2\alpha-1)\theta_H^2 + (1-\alpha)\theta_L^2 + 2\alpha\rho\sigma^2)}. \tag{25}$$

The resulting low type's certainty equivalent is

$$\widehat{U}_{L,PCL}^B = \frac{1}{2}(1-w_H^B)w_H^B\beta^2\theta_H^2 + \frac{{w_H^B}^2}{4}(\beta^2\theta_L^2 - 2\rho\sigma^2), \tag{26}$$

with w_H^B given by Equation (25).

Case 2. (PCH) *is non-binding.*

In this case, $\max(\widehat{U}_{L,CS}^B, \widehat{U}_{L,PCL}^B) \neq \widehat{U}_{L,PCL}^B$, since at least one of the participation contraints is binding in the good firm's best response. If the maximum is $\widehat{U}_{L,CS}^B > \widehat{U}_{L,LCS}^B$ (i.e., LCS is not IE), then $\widehat{U}_L^G < \widehat{U}_{L,CS}^B$ may hold in equilibrium as it may not give rise to a profitable deviation which attracts the high type: $\widehat{U}_L^G$ can be lowered to the point where either the low type receives her expected output, or where the high type can be profitably attracted, which then requires that *(PCH)* is binding.

Summing up so far, any $\widehat{U}_L^G \geq \underline{U}_L^B$ rules out that the bad firm can profitably attract any type, whereas any $\widehat{U}_L^G < \underline{U}_L^B$ entails that the bad firm can profitably attract at least one type. Since the good firm attracts both types in equilibrium, the latter case does not constitute an equilibrium, while the former case can constitute an equilibrium provided the good firm is willing to bid and provided that best responses are not weakly dominated in the sense of Assumption 1.

Therefore, consider now the good firm's best response as given by Equation (6). The good firm's best response to $\widehat{U}_L^B > \widehat{U}_{L,\max}^B$, where the bad firm offers the low type even more than her expected output in the *good* firm, is to not bid for the low type. Thus, $\widehat{U}_L^B > \widehat{U}_{L,\max}^B$ cannot hold in equilibrium, as we know from Lemma 3 that the good firm hires both types in equilibrium. For $\widehat{U}_L^B \leq \widehat{U}_{L,\max}^B$ and given the binding *(PCL)*, the good firm attracts the low type, for it will otherwise just forego the profit derived from her.

Similarly, the good firm will not offer the high type more than her expected output. Given that the contract for the high type in the bad firm is inefficient and offers exactly her output, this case is subsumed by $\widehat{U}_{L,\max}^B$. This proves that the good firm's best response to any offer below $\widehat{U}_{L,\max}^B$ is to attract both types.

The greatest certainty equivalent offered to the low type such that the high type receives her output is determined by the case when both $w_H^B = w_H^{B,sb}$ and *(ICCLB)* is binding, which is given by

$$\widehat{U}_{L,\text{no imi.}}^B := U_L^B(F_H^{B,sb}, w_H^{B,sb}) = \frac{\beta^4\theta_H^4(\beta^2\theta_L^2 + 2\rho\sigma^2)}{4(\beta^2\theta_H^2 + 2\rho\sigma^2)^2}. \tag{27}$$

As Assumption 1 excludes weakly dominated strategies, the bad firm will not offer the low type more than $\widehat{U}_{L,\text{no imi.}}^B$if $\widehat{U}_{L,\text{no imi.}}^B > \widehat{U}_{L,LCS}^B$, as the only reason to offer the low type more than her expected output is to keep her from imitating. And as a non-binding *(ICCLB)* implies that this is not necessary, this is weakly dominated.

Finally, the offers by the bad firm must be such that the high type is offered her expected output by the bad firm, cf. Lemma 2. The highest certainty equivalent offered to the *low* type such that the high type receives her expected output fulfills the first-order condition

$$\frac{\partial}{\partial w_H^B}\left[\frac{1}{2}(1 - w_H^B)w_H^B\beta^2\theta_H^2 + \frac{{w_H^B}^2}{2}\left(\frac{\beta^2\theta_L^2}{2} - 2\rho\sigma^2\right)\right] = 0,$$

cf. Equations (21b) and (21c), which in turn yields $U_L^B(F_H^B, w_H^B) = \dfrac{\beta^4\theta_H^4}{4(\beta^2(2\theta_H^2 - \theta_L^2) + 2\rho\sigma^2)}$. It is easily shown that this expression is greater than $\widehat{U}_{L,\text{no imi.}}^B$, so that this case can be ignored.

Summing up, $\widehat{U}_L^G < \underline{U}_L^B$ implies that the bad firm has a profitable deviation, whereas $\widehat{U}_L^B > \overline{U}_L^B$ implies that either the good firm does not attract both types or that the bad firm's best response is weakly dominated. Any $\widehat{U}_L^B \in [\underline{U}_L^B, \overline{U}_L^B]$ fulfills the necessary conditions of Lemmas 2 and 3, and is also sufficient as the offered contracts are mutual best responses. □

Proof of Proposition 6. We show that $\dfrac{\partial\Delta\widehat{U}^B}{\partial\beta} \geq 0$, for both $\widehat{U}_L^B = \underline{U}_L^B$ and $\widehat{U}_L^B = \overline{U}_L^B$ in $\Delta\widehat{U}^B$. By Equations (21a)-(21c) we have

$$\Delta\widehat{U}^B(\beta) = U_H^B(0, w_H^{B,*}(\beta)) - U_L^B(0, w_H^{B,*}(\beta)) = \frac{w_H^{B,*}(\beta)^2}{4}\beta^2(\theta_H^2 - \theta_L^2),$$

with $w_H^{B,*}(\beta)$ given according to Equation (24) (observe that $\widehat{U}_L^G$ depends on β). Hence,

$$\frac{\partial}{\partial\beta}\Delta\widehat{U}^B = \frac{w_H^{B,*}(\beta)}{2}\left\{\beta w_H^{B,*\prime}(\beta) + w_H^{B,*}(\beta)\right\}\beta(\theta_H^2 - \theta_L^2),$$

and it is sufficient to show that $w_H^{B,*\prime}(\beta) \geq 0$.

Case 1: $U_L^B = \underline{U}_L^B$.

Since *(PCH)* is binding, $\underline{U}_L^B = \max(\widehat{U}_{L,CS}^B, \widehat{U}_{L,PCL}^B)$. Suppose first that cases do not change at β, and consider the following three cases: $\underline{U}_L^B = \widehat{U}_{L,CS}^B = \widehat{U}_{L,LCS}^B$, $\underline{U}_L^B = \widehat{U}_{L,CS}^B > \widehat{U}_{L,LCS}^B$ and $\underline{U}_L^B = \widehat{U}_{L,PCL}^B$ in turn.

Assume first that $\underline{U}_L^B = \widehat{U}_{L,LCS}^B = \dfrac{\beta^4\theta_L^4}{4(\beta^2\theta_L^2 + 2\rho\sigma^2)}$. Inserting this expression into Equation (24) and taking the derivative, we obtain

$$\begin{aligned} w_H^{B,*\prime}(\beta) &= \frac{4\beta\rho\sigma^2\left(\beta^6(\theta_H^2\theta_L^6 - \theta_H^4\theta_L^4) + \beta^2(\theta_H^2\theta_L^2\sqrt{\beta^8\theta_L^4(\theta_H^2 - \theta_L^2)^2 + 2\rho\sigma^2(\text{pos. terms})})\right)}{\text{positive terms}} \\ &\quad + \text{positive terms} \\ &\geq 0. \end{aligned}$$

If $\underline{U}_L^B = \widehat{U}_{L,PCL}^B$, by definition $\dfrac{\partial}{\partial\beta}\Delta\widehat{U}^B = 0$.

Last, consider the case where $\underline{U}_L^B = \widehat{U}_{L,CS}^B$, assuming that $\widehat{U}_{L,CS}^B > \widehat{U}_{L,LCS}^B$. Let $\widehat{U}_{H,CS}^B$ be the minimum utility to be offered to the high type by the bad firm from the cross-subsidy strategy where the low type receives some of the high type's expected output, see the proof of Proposition 5, in particular, Equation (22). By definition, we have $U_H^B(F_H^B(w_H^{B,*}), w_H^{B,*}) = \widehat{U}_{H,CS}^B$, and get

$$\frac{\partial}{\partial\beta}U_H^B = \frac{\partial}{\partial\beta}\widehat{U}_{H,CS}^B. \tag{28}$$

To deduce that $w_H^{B,*} \geq 0$ requires making each side of Equation (28) explicit. In the cross-subsidy contract from Equation (22), $w_L^{B,*} = w_L^{B,sb}$ and *(ICCLB)* is binding. Then, the problem that solves the cross-subsidy contract reduces to one variable, the optimal piece rate for the high type, denoted by w^*. As w^* solves the first-order condition $\dfrac{\partial}{\partial w}\,\widehat{U}_{H,CS}^B(w^*,\beta) = 0$, we get

$$\widehat{U}_{H,CS}^B{}'(w^*(\beta),\beta) = \frac{\partial}{\partial\beta}\widehat{U}_{H,CS}^B(w^*,\beta) + w^{*\prime}(\beta)\underbrace{\frac{\partial}{\partial w}\widehat{U}_{H,CS}^B(w^*,\beta)}_{=0},$$

and

$$\begin{aligned}
\frac{\partial}{\partial\beta}\widehat{U}^B_{H,CS}(w^*,\beta) &= \alpha(1-w^*)w^*\beta\theta_H^2 \\
&\qquad + (1-\alpha)\left\{(1-w_L^{B,sb})w_L^{B,sb}\beta\theta_L^2 - \frac{w^{*2}-w_L^{B,sb2}}{2}\beta\theta_L^2\right\} + \frac{w^{*2}}{2}\beta\theta_H^2 \\
&= \frac{2}{\beta}\left\{\widehat{U}^B_{H,CS} - (1-\alpha)\frac{w^{*2}-w_L^{B,sb2}}{2}\rho\sigma^2 + \frac{w^{*2}}{2}\rho\sigma^2\right\} \qquad (29) \\
&= \frac{2}{\beta}\left\{\widehat{U}^B_{H,CS} + (1-\alpha)\frac{w_L^{B,sb2}}{2}\rho\sigma^2 + \alpha\frac{w^{*2}}{2}\rho\sigma^2\right\}
\end{aligned}$$

On the other hand, with the piece rate offered in equilibrium,

$$\begin{aligned}
\frac{\partial}{\partial\beta}U_H^B(w_H^{B,*}(\beta),\beta) &= \frac{\partial}{\partial\beta}U_H^B(w_H^{B,*},\beta) + \ w_H^{B,*\prime}(\beta)\frac{\partial}{\partial w}U_H^B(w_H^{B,*},\beta) \\
&= \frac{2}{\beta}\left\{U_H^B + \frac{w_H^{B,*2}}{2}\rho\sigma^2\right\} + w_H^{B,*\prime}(\beta)\left\{\frac{\beta^2\theta_H^2}{2} - \frac{w_H^{B,*}\beta^2\theta_H^2}{2} - w_H^{B,*}\rho\sigma^2\right\}. \qquad (30)
\end{aligned}$$

To show that $w_H^{B,*\prime}(\beta)\geq 0$, recall first that because of Equation (28), the expressions (29) and (30) must be identical. Observe further that $\frac{\partial}{\partial w}U_H^B(w,\beta)\leq 0$ for $w\geq w_H^{B,sb}$, the high type's optimal contract when she receives her output. This implies that the second term of Equation (30) is negative if $w_H^{B,*\prime}$ is positive and vice versa. If we show that $w_H^{B,*}>w^*$, which implies that the first term of Equation (30) is greater than Equation (29), then it follows directly by the equality of Equation (29) and Equation (30) that $w_H^{B,*\prime}$ is positive. But to see that $w_H^{B,*}>w^*$ observe that $U_H^B(F_H^B(w^*),w^*)>U^B_{H,CS}$ since the agent receives her full output in the first case, while she receives less in the cross-subsidy case.

It remains to observe that the above results also cover the cases where the case distinction of U_L^B switches due to the continuity of all variables involved.

Case 2: $U_L^B=\overline{U}_L^B$.

We now show that $w_H^{B,*\prime}(\beta)\geq 0$ for $U_L^B=\overline{U}_L^B$. Again, we need to consider the two candidates for $\overline{U}_L^B$ separately. For $\overline{U}_L^B=\widehat{U}_{L,\text{no imi.}}$, by definition $w_H^{B,*}=w_H^{B,sb}=\frac{\beta^2\theta_H^2}{\beta^2\theta_H^2+2\rho\sigma^2}$, which is increasing in β. For $\overline{U}_L^B=\widehat{U}_{L,\max}$, we distinguish two cases: First, in Regions 2 and 3, the piece rate for the low type is second-best, so that $\widehat{U}_{L,\max}=\frac{\theta_L^4}{4(\theta_L^2+2\rho\sigma^2)}$. In this case, $w_H^{B,*}$ is given by Equation (24) with $\widehat{U}_L^G$ replaced by $\widehat{U}_{L,\max}$, which is a constant that does not depend on β. One can then easily show that $w_H^{B,*}$ is increasing in β. Second, in Region QM(b), the piece rate for the low type is distorted and given by $w_L^* = \beta w_H^{B,*}$, which is easily derived from the expression for w_L^* in Region QM(b) given in Proposition 1 and the binding *(IC-CLB)*. Hence, when the low type receives her full output from the good firm, then $\widehat{U}_L = (1-\beta w_H^{B,*})\beta w_H^{B,*}\frac{\theta_L^2}{2} + \frac{(\beta w_H^{B,*})^2}{4}(\theta_L^2 - 2\rho\sigma^2)$, and plugging this into Equation (24) yields $w_H^{B,*} = \frac{\beta(\beta\theta_H^2 - \theta_L^2)}{\beta^2(\theta_H^2-\theta_L^2)+(1-\beta^2)\rho\sigma^2}$. This expression is non-negative if $\beta\theta_H^2 \geq \theta_L^2$, and, in particular, we have that $w_H^{B,*} \geq w_H^{B,sb} > 0$, because in the bad firm, the contract for the high type is distorted. Hence, it is sufficient to analyze $w_H^{B,*\prime}(\beta)$ under the condition that $\beta\theta_H^2 \geq \theta_L^2$. We have

$$\begin{aligned} w_H^{B,*\prime}(\beta) &= \frac{\beta^2\theta_L^2(\theta_H^2-\theta_L^2)+\rho\sigma^2(2\beta\theta_H^2-\theta_L^2-\beta^2\theta_L^2)}{(\beta^2(\theta_H^2-\theta_L^2-\rho\sigma^2)+\rho\sigma^2)^2} \\ &\geq \frac{\beta^2\theta_L^2(\theta_H^2-\theta_L^2)+\rho\sigma^2(1-\beta^2)\theta_L^2}{(\beta^2(\theta_H^2-\theta_L^2-\rho\sigma^2)+\rho\sigma^2)^2} \geq 0, \end{aligned}$$

and the claim follows.

$\square$

A.4 Including horizontal differentiation

To enhance readability, we omit the index k of the firm under consideration. As firms only differ in β, no confusion should arise.

Proof of Lemma 4. 1. Consider an interior best response (w_H^*, U_H^*) and (w_L^*, U_L^*) and suppose that $w_H^* < w_H^{sb}$. Let $x_H^* \in (0,1)$ be the location of an H-type agent whose participation constraint is binding, i.e., $x_H^* = \frac{1}{2} + \frac{U_H - \widehat{U}_H}{2t}$. Now consider U_H such that $U_H - x_H^* t = \widehat{U}_H - (1-x_H^*)t$, i.e., (w_H^{sb}, U_H) satisfies PCH for H-type agents located on $[0, x_H^*]$. Observe that ICCH continues to hold as H-type agents (at any given location) receive the same utility under (U_H, w_H^{sb}) and (U_H^*, w_H^*). Recall from the previous section that $v(w_H)$ is increasing in $w_H < w_H^{sb}$. Hence, under (U_H, w_H^{sb}) the same set of agents located in $[0, x_H^*]$ participates and generates a larger profit for the firm (if H-type and only H-type agents choose (U_H, w_H^{sb})). As U_L^* (and thereby x_L^*) is left unaltered, we are left to show that (U_H, w_H^{sb}) and (w_L^*, F_L^*) satisfy ICCL (and L-type agents therefore choose (w_L^*, F_L^*) and receive the same utility as in the original menu of contracts). To see this, observe that the utility for the L type agent who signs the H type's contract is $U_H - \frac{(w_H^{sb})^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$, and

$$\begin{aligned} & U_H - \frac{(\beta w_H^{sb})^2}{4}(\theta_H^2 - \theta_L^2) \\ & < U_H^* - \frac{(\beta w_H^*)^2}{4}(\theta_H^2 - \theta_L^2) \end{aligned}$$

which is the utility enjoyed by an L type agent who signs the original (*) contract (here, we used the definition of U_H (i.e., $U_H = U_H^*$) and $w_H^* < w_H^{sb}$). Hence, imitation incentives are strictly smaller under (w_H^{sb}, F) and ICCL is satisfied whenever it was satisfied in the original menu. This contradicts the optimality of (w_H^*, F_H^*). The proof for $w_L^* \leq w_L^{sb}$ proceeds analogously.

2. Suppose there is an optimal menu of contracts (w_H^*, U_H^*) and (w_L^*, U_L^*) with $w_H^* > w_H^{sb}$. We proceed in three steps. (i) We show that ICCH is not binding whenever ICCL is binding, (ii) we show that ICCL is binding, and (iii) we argue that $w_L^* = w_L^{sb}$.

(i) Suppose that ICCL is binding, i.e., $U_L = U_H - \frac{(w_H^*)^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$. Then, we can rewrite

$$U_H - U_L = \frac{(w_H^*)^2}{4}\beta^2(\theta_H^2 - \theta_L^2) > \frac{(w_L^*)^2}{4}\beta^2(\theta_H^2 - \theta_L^2).$$

The strict inequality follows from $w_H^* > w_H^{sb} > w_L^{sb} \geq w_L^*$ (see Part 1 of the Lemma). Hence, ICCH is not binding whenever ICCL is binding.

(ii) For given U_H and U_L, ICCL reads $U_H - U_L \leq (w_H^*)^2\beta^2(\theta_H^2 - \theta_L^2)$. As $v_H(w_H)$ and thereby Π_H for a given U_H is decreasing in $w_H^* > w_H^{sb}$, it is optimal for the firm to choose the minimal w_H that satisfies ICCH and ICCL. But as ICCH is automatically satisfied whenever ICCL is binding, an optimal w_H^* will be such that ICCL is binding.

(iii) Recall from Part 1 that $w_L^* \leq w_L^{sb}$. As demonstrated in (i) and (ii), ICCL is binding and ICCH is non-binding for any w_L^*. But as $v_L(w_L)$ (and thereby Π_L for a given U_H and U_L) is maximized by w_L^{sb}, it follows that $w_L^* = w_L^{sb}$.

□

Proof of Proposition 7. (i) To simplify expressions, we consider U_L and ΔU as a firm's choice variables (again dropping index k if no confusion can arise) rather than U_i and w_i as in (8). As Lemma 4 indicates that w_i is either second best or w_i^2 is proportional to ΔU this is an equivalent formulation of the firm's optimization program. We prove that $\frac{\partial \Delta U}{\partial \Delta \widehat{U}} > 0$ for quasi-competitive contracts. The proof for quasi-monopsonistic and second-best contracts proceeds analogously. In the same way, we can also demonstrate that $\frac{\partial U_L}{\partial \Delta \widehat{U}} > 0$, $\frac{\partial U_L}{\partial \widehat{U}_L} > 0$, and $\frac{\partial \Delta U}{\partial \widehat{U}_L} > 0$.

For a quasi-competitive contract, $w_L^* = w_L^{sb}$ and $\Delta U = \frac{w_H^2 \beta^2}{4}(\theta_H^2 - \theta_L^2)$ induce the following FOCs for ΔU and U_L:

$$F_{U_L} \equiv \alpha\Pi_H - \alpha(t + U_H - \widehat{U}_H) + (1-\alpha)\Pi_L - (1-\alpha)(t + U_L - \widehat{U}_L) = 0$$

$$F_{\Delta U} \equiv \alpha\Pi_H + \alpha(t + U_H - \widehat{U}_H)(v_H'(w_H)\frac{dw_H}{d\Delta U} - 1) = 0$$

The associated Hessian matrix satisfies $\det(H) > 0$ whenever $(1-\alpha) + \alpha v_H'(w_H)\frac{dw_H}{d\Delta U} \geq 0$ (details available on request). Observe that $(1-\alpha) + \alpha v_H'(w_H)\frac{dw_H}{d\Delta U} = (1-\alpha) + \alpha v_H'(w_H)\frac{1}{\beta^2(\theta_H^2 - \theta_L^2)\frac{w_H}{2}}$. Hence, this condition is satisfied if and only if

$$\alpha v'(w_H) + (1-\alpha)\frac{w_H}{2}\beta^2(\theta_H^2 - \theta_L^2) \geq 0.$$

As indicated by Proposition 2, this condition is satisfied if $\alpha < \alpha_{LCS}$.

By Cramer's rule, $\frac{\partial \Delta U}{\partial \Delta \widehat{U}} = -\frac{\det(H_{\Delta U, \Delta \widehat{U}})}{\det(H)}$ where $H_{\Delta U, \Delta \widehat{U}}$ is the matrix with the first row being $(\frac{\partial F_{U_L}}{\partial U_L}, \frac{\partial F_{U_L}}{\partial \Delta \widehat{U}})$ and the second row being $(\frac{\partial F_{\Delta U}}{\partial U_L}, \frac{\partial F_{\Delta U}}{\partial \Delta \widehat{U}})$. $\det(H_{\Delta U, \Delta \widehat{U}}) < 0$ (such that $\frac{\partial \Delta U}{\partial \Delta \widehat{U}} > 0$) whenever $\alpha v'(w_H) + (1-\alpha)\frac{w_H}{2}\beta^2(\theta_H^2 - \theta_L^2) \geq 0$ (detailed computations available on request).

(ii) For $\Delta \widehat{U} = 0$ and $\widehat{U}_L = \overline{U}$, the best response is the monopsony menu with $\Delta U = \Delta U^m$ and $U_L = U_L^m$. For $\Delta \widehat{U} = \Delta U^c \equiv \frac{(w_H^c)^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ and $\widehat{U}_L = U_L^c$, the best response is $\Delta U = \Delta U^c > \Delta U^m$ and $U_L = U_L^c > U_L^m$. As the best responses ΔU and U_L are continuous and strictly monotone increasing in $\Delta \widehat{U}$ and $\widehat{U}_L$ (see Part (i)) there is a unique $\Delta \widehat{U}_1$ such that the best response is $\Delta U_1 = \frac{(w_L^{sb})^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ and a unique $\Delta \widehat{U}_2$ such that the best response is $\Delta U_2 = \frac{(w_H^{sb})^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$.

□

Proof of Lemma 5. Consider firm k and denote firm $\bar{k}$'s contract offer by $\widehat{()}$. We proceed as follows. We will first show that both firms will hire some agents as long as β is sufficiently large and second prove that no type of agent will be excluded under these conditions.

1. Suppose $\widehat{()}$ is such that $\widehat{U}_i < \overline{U} + t$ for at least one type $i = H, L$, i.e., agents of type i that are sufficiently close to firm k prefer the outside option to the other firm's offer. As long as $v_i(w_i^{sb}) > \overline{U}$, the firm receives a positive profit from offering $(w_i^{sb}, \overline{U} + \epsilon)$. As $\overline{U}$ is normalized to zero, $v_i(w_i^{sb}) > \overline{U}$ always holds for the good firm and holds for the bad firm on an open neighborhood of $\beta = 1$..

2. Suppose $\widehat{()}$ is such that $\widehat{U}_i \geq \overline{U} + t$ for all types i, i.e., for all agents, the offer of firm $\bar{k}$ resembles the outside option. In equilibrium, firm $\bar{k}$ only offers this menu if it generates positive expected profits. If firm k offers the same menu of contracts, half of the agents of both types will be attracted by firm k. If firm k is the good firm or β is sufficiently close to 1, this also generates a positive profit for firm k. Hence, the good firm and the bad firm for β in an open neighborhood of $\beta = 1$ will always hire a positive mass of agents in any equilibrium. It remains to discuss conditions under which both firms hire agents of both types.

3. For the good firm and for the bad firm on a open neighborhood of $\beta = 1$, there is no incentive to exclude H types in any equilibrium. To see this, suppose firm k does not hire H types but a positive mass of L types and receives positive profits, i.e., $U_L > \widehat{U}_L - t$, $U_L > \overline{U}$, $U_L < v_L(w_L)$, and (by optimality) $w_L = w_L^{sb}$. We have to distinguish three cases. Case 1: Suppose firm $\bar{k}$ offers a quasi-monopsony contract, i.e., $\widehat{U}_H = \widehat{U}_L + \frac{\widehat{w}_L^2}{4}\widehat{\beta}^2(\theta_H^2 - \theta_L^2)$. Then, an H-type agent receives $\widetilde{U}_H = U_L + \frac{(w_L^{sb})^2}{4}\beta^2(\theta_H^2 - \theta_L^2) > \widehat{U}_L - t + \frac{(w_L^{sb})^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ from accepting the contract offered by firm k to L types. Whenever $\beta w_L^{sb} \geq \widehat{\beta}\widehat{w}_L$, this is at least $\widehat{U}_H - t$ such that a positive mass of H types prefers this contract to the contract menu offered by firm $\bar{k}$ (and by $U_L > \overline{U}$ to the outside option). Hence, a positive mass of H types signs the contract with firm k and H types are not excluded on an open neighborhood of $\beta = 1$ Case 2: Suppose firm $\bar{k}$ offers a quasi-competitive or second-best contract with $\widehat{\Pi}_H > 0$ (which holds whenever

$t > 0$) and $U_L > \widehat{U}_L$. Then, firm k generates positive profits from offering $(\widehat{w}_H, \widehat{U}_H)$. To see this, observe that L types prefer (w_L, U_L) as $U_L > \widehat{U}_L \geq \widehat{U}_H - \frac{\widehat{w}_H^2}{4}\widehat{\beta}^2(\theta_H^2 - \theta_L^2)$ where the last inequality follows from ICCL. Whenever $\beta \geq \widehat{\beta}$, this (weakly) exceeds $\widehat{U}_H - \frac{\widehat{w}_H^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ and there is an open neighborhood of $\beta = 1$ such that $U_L \geq U_H - \frac{w_H^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ if $1 = \widehat{\beta} > \beta$. Case 3: Suppose firm $\bar{k}$ offers a quasi-competitive or second-best contract with $\widehat{\Pi}_H > 0$ and $U_L \leq \widehat{U}_L$. Then, firm k generates positive profits from offering $(\widehat{w}_H, U_L + \frac{\widehat{w}_H^2}{4}\beta^2(\theta_H^2 - \theta_L^2))$. This contract leaves L types indifferent to (w_L, U_L) as ICCL is binding. But as firm k attracts a positive mass of L types, $U_L > \widehat{U}_L - t$ such that $U_H = U_L + \frac{\widehat{w}_H^2}{4}\beta^2(\theta_H^2 - \theta_L^2) > \widehat{U}_L - t + \frac{\widehat{w}_H^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ which (weakly) exceeds $\widehat{U}_H - t$ whenever $\beta \geq \widehat{\beta}$ by ICCL in firm $\bar{k}$. Moreover, $U_L > \widehat{U}_L - t$ also implies that there is an open neighborhood of $\beta = 1$ such that $U_H > \widehat{U}_H - t$ for $1 = \widehat{\beta} > \beta$.

4. Finally, for the good firm and for the bad firm on a open neighborhood of $\beta = 1$, there is no incentive to exclude L types in any equilibrium. The incentive to exclude L types is maximal in monopsony (i.e., $t \to \infty$), as the surplus generated by L types is decreasing in t and H types have an imitation incentive in this case.In the monopsony case, firm k maximizes $\Pi = \alpha(v_H(w_H) - U_H) + (1-\alpha)(v_L(w_L) - U_L)$ s.t. the binding ICCH, i.e., $U_H = U_L + \frac{w_L^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$. This yields as a FOC: $(1-\alpha)v_L'(w_L) = \alpha\frac{w_L}{2}\beta^2(\theta_H^2 - \theta_L^2)$. Observe that this implies that the optimal w_L is decreasing in α and $w_L = 0$ for $\alpha = 1$ as $v'(w_L)$ is bounded for $w_L \leq w_L^{sb}$. Excluding L types is optimal if the expected profit with the optimal contract for low types is below the rent savings for high types, i.e.,

$$(1-\alpha)\Pi_L \leq \alpha\frac{w_L^2}{4}\beta^2(\theta_H^2 - \theta_L^2). \quad (*)$$

As Π_L is bounded for $w_L \leq w_L^{sb}$ and $w_L = 0$ for $\alpha = 1$, both sides of $(*)$ are zero for $\alpha = 1$. Now observe that the slope of the LHS (with respect to α) is $-\Pi_L + (1-\alpha)v_L'(w_L)\frac{dw_L}{d\alpha}$ and the slope of the RHS is $\frac{w_L^2}{4}\beta^2(\theta_H^2 - \theta_L^2) + \alpha\frac{w_L}{2}\beta^2(\theta_H^2 - \theta_L^2)\frac{dw_L}{d\alpha}$.Then, the FOC implies that the LHS decreases more steeply in α than the RHS (and coincide at $\alpha = 1$). Hence, $(*)$ is never satisfied as a strict inequality and excluding L types because of rent savings never resembles a strictly better reply. □

Proof of Lemma 6. We need to rule out cases where for at least one type i and at least one firm $U_i > \widehat{U}_i + t$.

Case 1: Suppose $U_i > \widehat{U}_i + t$ for $i = H, L$ in a profit maximizing contract menu. Then, the firm could lower U_i for both types without altering incentive compatibility and individual rationality for both types on all locations. A contradiction to profit maximization.

Case 2: Suppose $U_H - \widehat{U}_H \leq t < U_L - \widehat{U}_L$, i.e., $x_L = 1$ and $x_H \leq 1$. According to Lemma 5, there is an open neighborhood of $\beta = 1$ such that no firm excludes any of the two types. Hence, $\widehat{U}_L > \overline{U}$ such that $U_L > \overline{U}$. The firm's objective is therefore to maximize $\alpha x_H \Pi_H + (1-\alpha)\Pi_L$ subject to $U_H \geq U_L + \frac{w_L^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ (ICCH, multiplier

μ_H) and $U_L \geq U_H - \frac{w_H^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ (ICCL, multiplier μ_L). Maximizing with respect to U_L yields the first order condition $-(1-\alpha) - \mu_H + \mu_L = 0$. As ICCL is binding in Case 2 (otherwise $U_L > \widehat{U}_L + t$ cannot be optimal), we get $\mu_L > 0$ and $\mu_H = 0$ such that $\mu_L = (1-\alpha)$. Furthermore, the binding ICCL, a satisfied ICCL of the offer by the other firm, and $U_H - \widehat{U}_H \leq t < U_L - \widehat{U}_L$ implies

$$\frac{w_H^2}{4}\beta^2(\theta_H^2 - \theta_L^2) = U_H - U_L < \widehat{U}_H - \widehat{U}_L \leq \frac{\widehat{w}_H^2}{4}\widehat{\beta}^2(\theta_H^2 - \theta_L^2)$$

or $\beta w_H < \widehat{\beta}\widehat{w}_H$. Then, the first order condition for w_H reads

$$\alpha x_H v_H'(w_H) + (1-\alpha)\frac{w_H}{2}\beta^2(\theta_H^2 - \theta_L^2) = 0\,(*).$$

If LCS for $t = 0$ with piece-rates w_i^c is IE, i.e., $\alpha v_{HH}'^c) + \frac{w_H^c}{2}\beta^2(\theta_H^2 - \theta_L^2) \geq 0$, we get $\alpha x_H v'(w_H) + (1-\alpha)\frac{w_H}{2}\beta^2(\theta_H^2 - \theta_L^2) > 0$ for all $w_H < w_H^c$ and $x_H \leq 1$ (see proof of Proposition 2). Hence, cornering is inferior in Case 2.

Case 3: Suppose $U_L - \widehat{U}_L \leq t < U_H - \widehat{U}_H$, i.e., $x_H = 1$ and $x_L \leq 1$. As no type is excluded by the other firm, $\widehat{U}_L > \overline{U}$ such that $U_L > \overline{U}$. The firm's objective is therefore to maximize $\alpha\Pi_H + (1-\alpha)x_L\Pi_L$ subject to $U_H \geq U_L + \frac{w_L^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ (ICCH, multiplier μ_H) and $U_L \geq U_H - \frac{w_H^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ (ICCL, multiplier μ_L). Maximizing with respect to U_L yields the first order condition $-\alpha + \mu_H - \mu_L = 0$. As ICCH is binding in Case 3 (otherwise cornering H types would be suboptimal), we get $\mu_H > 0$ and $\mu_L = 0$ such that $\mu_H = \alpha$. Furthermore, the binding ICCH, a satisfied ICCL of the offer by the other firm, and $U_L - \widehat{U}_L \leq t < U_H - \widehat{U}_H$ implies

$$\frac{w_L^2}{4}\beta^2(\theta_H^2 - \theta_L^2) = U_H - U_L > \widehat{U}_H - \widehat{U}_L \geq \frac{\widehat{w}_L^2}{4}\widehat{\beta}^2(\theta_H^2 - \theta_L^2)$$

or $\beta w_L \geq \widehat{\beta}\widehat{w}_L \geq \beta w_L^m$. Then, the first order condition for w_L reads

$$(1-\alpha)x_L v_L'(w_L) - \alpha\frac{w_L}{2}\beta^2(\theta_H^2 - \theta_L^2) = 0\,(*).$$

In the monopsony case (see above), the corresponding first order condition reads $(1-\alpha)v_L'^m) = \alpha\frac{w_L^m}{2}\beta^2(\theta_H^2 - \theta_L^2)$. Inserting the first order condition for monopsony into $(*)$ yields $x_L = \frac{v'(w_L^m)}{v'(w_L)}\frac{w_L}{w_L^m} > 1$. As $w_L \geq w_L^m$ we get a contradiction. Hence, cornering H types is never optimal in Case 3 which concludes the proof.

□

Proof of Proposition 8. We proceed in two steps: First, we show that if cornering is not feasible (i.e., $U_i^k \leq U_i^{\bar{k}} + t$) and β is sufficiently large to admit an interior solution, there is always a pure strategy equilibrium C^*. I.e., C^* is an equilibrium if and only if cornering is not a profitable deviation. Second, we demonstrate that cornering is not a best response against C^* if and only if C^* is interim efficient, i.e., if firm $\bar{k}$ offers the menu in C^*, offering C^* is Pareto efficient.

Step 1: If cornering is not feasible, best responses U_L^k and ΔU^k are continuous and strictly monotone increasing functions of $\widehat{U}_L^k$ and $\Delta\widehat{U}^k$ (see Proposition 7). As the strategy space is a lattice, this implies by Milgrom and Roberts (1994, Theorem 3) the existence of a pure strategy equilibrium C^*. It remains to show that if β is sufficiently large to render exclusion unprofitable (see Lemma 5), C^* constitute an equilibrium if and only if cornering is not a profitable deviation against C^*.

$\Leftarrow$ If cornering is not a profitable deviation against C^*, C^* remains an equilibrium if cornering is feasible.

$\Rightarrow$ If cornering is a profitable deviation against C^*, C^* is not a pure strategy equilibrium if cornering is feasible.

But if mutual best replies without cornering do not constitute a pure strategy equilibrium as soon as cornering is permitted, any pure strategy equilibrium of the game with cornering being feasible has to involve a cornering contract. But a cornering contract can never be part of a pure strategy equilibrium. To see this recall that cornering H types is never optimal (see the proof of Lemma 6) and cornering L types is only optimal for firm k if it offers a quasi-competitive contract and cornering allows to reduce imitation incentives for L types to reduce piece rate distortions for H types. Now suppose firm $\bar{k}$ attracts all L types, i.e., $U_L \leq \widehat{U}_L - t$, and firm k attracts some H types, i.e., $U_H > \widehat{U}_H - t$. Then, $U_H - U_L > \widehat{U}_H - \widehat{U}_L$. As firm $\bar{k}$ offers a quasi-competitive contract (see above), ICCL is binding and $\widehat{U}_H - \widehat{U}_L = \frac{\widehat{w}_H^2}{4}\widehat{\beta}^2(\theta_H^2 - \theta_L^2)$. Then, ICCL for contracts offered by firm k implies $\beta w_H > \widehat{\beta}\widehat{w}_H$. If $1 = \widehat{\beta} \geq \beta$, this implies $w_H > \widehat{w}_H$ and the gain from cornering $-\alpha v'(w_H) - (1-\alpha)\frac{w_H}{2}\beta^2(\theta_H^2 - \theta_L^2)$ (which is decreasing in β and increasing in w_H) is larger for firm k than for firm $\bar{k}$. This result is unaltered if $1 = \beta \geq \widehat{\beta}$ and $\widehat{\beta}$ is sufficiently close to 1. Hence, if the good firm (or the bad firm for β sufficiently close to 1) corners, the other firm has an incentive to not exclude L types. Then, cornering cannot be part of an equilibrium strategy and the sub-optimality of cornering against C^* is necessary and sufficient for the existence of a pure strategy equilibrium if β is sufficiently close to 1.

Step 2: We are left to show that cornering against C^* is not a profitable deviation if and only if C^* is interim efficient. $\Leftarrow$ As in the proof of Lemma 6, if C^* is interim efficient, a cornering strategy cannot generate more surplus than C^* and cannot constitute a profitable deviation.

$\Rightarrow$ If cornering is a profitable deviation against C^*, (9) does not hold for w_H^* and C^* is not a best response against C^*.

$\square$

Proof of Proposition 9. According Milgrom and Roberts (1994, Theorem 3) it suffices to show that best responses are strictly monotone decreasing in t and strictly monotone increasing in β. This can be established for interim efficient C^* by routine computations using Cramer's rule as in the proof of Proposition 7. Consider, e.g., $\frac{d\Delta U}{dt}$ and recall from the proof of Proposition 7 the first order conditions for ΔU and U_L:

$$F_{U_L} \equiv \alpha\Pi_H - \alpha(t + U_H - \widehat{U}_H) + (1-\alpha)\Pi_L - (1-\alpha)(t + U_L - \widehat{U}_L) = 0$$

$$F_{\Delta U} \equiv \alpha\Pi_H + \alpha(t + U_H - \widehat{U}_H)(v'_H(w_H)\frac{dw_H}{d\Delta U} - 1) = 0$$

By Cramer's rule $\frac{d\Delta U}{dt} = -\frac{\det(H_{\Delta U,t})}{\det(H)}$ where $H_{\Delta U,t}$ is the matrix with the first row being $(\frac{\partial F_{U_L}}{\partial U_L}, \frac{\partial F_{U_L}}{\partial t})$ and the second row being $(\frac{\partial F_{\Delta U}}{\partial U_L}, \frac{\partial F_{\Delta U}}{\partial t})$. If C^* is interim efficient, i.e., $(1-\alpha)x^*_H v'(w^*_H) + (1-\alpha)\frac{w^*_H}{2}\beta^2(\theta^2_H - \theta^2_L) \geq 0$, it follows that $\det(H_{\Delta U,t}) > 0$ (such that $\frac{d\Delta U}{dt} < 0$).

□

References

ARNOTT, R., AND J. STIGLITZ (1991): "Equilibrium in competitive insurance markets with moral hazard," NBER Working Paper No. 3588.

ATTAR, A., T. MARIOTTI, AND F. SALANIÉ (2014a): "Multiple contracting in insurance markets: Cross-subsidies and quantity discounts," Toulouse School of Economics, Working Paper No IDEI-839.

——— (2014b): "Nonexclusive competition under adverse selection," *Theoretical Economics*, 9, 1–40.

BANNIER, C., E. FEESS, AND N. PACKHAM (2013): "Competition, Bonuses, and Risk-taking in the Banking Industry," *Review of Finance*, 17, 653–690.

BEBCHUK, L. A., A. COHEN, AND H. SPAMANN (2010): "The Wages of Failure: Executive Compensation at Bear Stearns and Lehman 2000-2008," *Yale Journal on Regulation*, 27, 257–282.

BÉNABOU, R., AND J. TIROLE (2016): "Bonus culture: Competitive pay, screening and multitasking," *Journal of Political Economy*, 124, 305–370.

BIGLAISER, G., AND C. MEZZETTI (1993): "Principals Competing for an Agent in the Presence of Adverse Selection and Moral Hazard," *Journal of Economic Theory*, 61, 302–330.

BISIN, A., AND P. GOTTARDI (1999): "Competitive equilibria with asymmetric information," *Journal of Economic Theory*, 87, 1–48.

DASGUPTA, P., AND E. MASKIN (1986): "The existence of equilibrium in discontinuous economic games, ii: Applications," *The Review of Economic Studies*, 53, 27–41.

GARRETT, D., R. GOMES, AND L. MAESTRI (2014): "Competitive screening under heterogeneous information," Mimeo, Toulouse School of Economics.

GARRETT, D., AND A. PAVAN (2012): "Managerial turnover in a changing world," *Journal of Political Economy*, 120, 879–925.

——— (2015): "Dynamic managerial compensation: A variational approach," *Journal of Economic Theory*, 159, 775–818.

HELLWIG, M. (1987): "Some Recent Developments in the Theory of Competition in Markets with Adverse Selection," *European Economic Review*, 31, 319–325.

HOLMSTROM, B., AND P. MILGROM (1987): "Aggregation and Linearity in the Provision of Intertemporal Incentives," *Econometrica*, 55, 303–328.

JULLIEN, B. (2000): "Participation constraints in adverse selection models," *Journal of Economic Theory*, 93, 1–47.

Lewis, T., and D. Sappington (1989): “Countervailing incentives in agency problems,” *Journal of Economic Theory*, 49, 294–313.

Maggi, G., and A. Rodriguez-Clare (1995): “On countervailing incentives,” *Journal of Economic Theory*, 66, 238–263.

Maskin, E., and J. Tirole (1992): “The Principal-Agent Relationship with an Informed Principal, II: Common Values,” *Econometrica*, 60, 1–42.

Milgrom, P., and J. Roberts (1994): “Comparing Equilibria,” *American Economic Review*, 84(3), 441–459.

Mimra, W., and A. Wambach (2011): “A game-theoretic foundation for the Wilson equilibrium in competitive insurance markets with adverse selection,” CESifo Working Paper No. 3412.

——— (2014): “New Developments in the Theory of Adverse Selection in Competitive Insurance,” *The Geneva Risk and Insurance Review*, 39, 136–152.

Miyazaki, H. (1977): “The Rat Race and Internal Labor Markets,” *Bell Journal of Economics*, 8, 394–418.

Moen, E., and A. Rosen (2005): “Performance Pay and Adverse Selection,” *Scandinavian Journal of Economics*, 107, 279–298.

Mussa, M., and S. Rosen (1978): “Monopoly and product quality,” *Journal of Economic Theory*, 18, 301–317.

Netzer, N., and F. Scheuer (2014): “A game theoretic foundation of competitive equilibria with adverse selection,” *International Economic Review*, 55, 399–422.

Packham, N. (2015): “Optimal contracts under adverse selection, moral hazard and type-dependent reservation utilities,” Working Paper, Frankfurt School of Finance & Management.

Picard, P. (2014): “Participating insurance contracts and the RothschildâStiglitz equilibrium puzzle,” *The Geneva Risk and Insurance Review*, 39, 153–175.

Pouyet, J., B. Salanié, and F. Salanié (2008): “On competitive equilibria with asymmetric information,” *The B.E. Journal of Theoretical Economics*, 8(1), 1–14.

Riley, J. (1979): “Informational Equilibrium,” *Econometrica*, 47, 331–359.

——— (2001): “Silver signals: Twenty-five years of screening and signaling,” *Journal of Economic Literature*, 39, 432–478.

Rothschild, M., and J. Stiglitz (1976): “Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information,” *Quarterly Journal of Economics*, 90, 629–649.

Salanié, B. (2005): *The Economics of Contracts*. MIT Press, 2 edn.

Schmidt-Mohr, U., and J. Villas-Boas (1999): "Oligopoly with asymmetric information: Differentiation in credit markets," *RAND Journal of Economics*, 30, 375–396.

——— (2008): "Competitive product lines with quality constraints," *Quantitative Marketing and Economics*, 6, 1–16.

Simon, L., and M. Stinchcombe (1995): "Equilibrium refinement for infinite normal-form games," *Econometrica*, 63, 1421–1443.

Spulber, D. (1989): "Product variety and competitive discounts," *Journal of Economic Theory*, 48, 510–525.

Stole, L. (1995): "Nonlinear pricing and oligopoly," *Journal of Economics and Management Strategy*, 4, 529–562.

——— (2007): "Price discrimination and competition," in *Handbook of Industrial Organization*, ed. by M. Armstrong, and R. Porter, vol. 3, chap. 34, pp. 2221–2299. Elsevier.

Sung, J. (2005): "Optimal contracts under adverse selection and moral hazard: A continuous-time approach," *Review of Financial Studies*, 18, 1021–1073.

Thanassoulis, J. (2013): "Industry structure, executive pay and short-termism," *Management Science*, 59, 402–419.

Tymula, A. (2014): "Competitive screening of a heterogeneous labor force and corporate teamwork attitude," Mimeo, New York University.

Wilson, C. (1977): "A Model of Insurance Markets with Incomplete Information," *Journal of Economic Theory*, 12, 167–207.

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Incentive schemes, private information and the double-edged role of competition for agents

Abstract
This paper examines the effect of imperfect labor market competition on the efficiency of compensation schemes in a setting with moral hazard and risk-averse agents, who have private information on their productivity. Two vertically differentiated firms compete for agents by offering contracts with fixed and variable payments. The superior firm employs both agent types in equilibrium, but the competitive pressure exerted by the inferior firm has a strong impact on contract design: For high degrees of vertical differentiation, i.e. low competition, low-ability agents are under-incentivized and exert too little effort. For high degrees of competition, high-ability agents are over-incentivized and bear too much risk. For a range of intermediate degrees of competition, however, agents' private information has no impact and both contracts are second-best. Interim efficiency of the least-cost separating allocation in the inferior firm is a sufficient condition for equilibrium existence. If this is violated, there can only be equilibria where the inferior firm "overbids", i.e. where it would not break even when attracting both agent types. Adding horizontal differentiation allows for pure-strategy equilibria even when there would be no equilibrium without overbidding in the pure vertical model, but equilibria with overbidding fail to exist.

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