ZeroSignVAR: A Zero and Sign Restriction Algorithm Implemented in MATLAB

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Abstract

ZeroSignVAR is a flexible MATLAB routine, which estimates vector autoregressions (VARs) in an Uhlig (1994) (Bayesian) fashion and identifies shocks using sign and/or zero restrictions. This vignette describes the application of ZeroSignVAR and the rich set of specification options that comes with the routine. The package delivers the set of structural models in the IRF, FEVD and HD representation.

Keywords: Sign Restrictions, Zero Restrictions, Structural Vector Autoregression, MATLAB

1 Introduction

Since the seminal work by Sims (1980), structural vector autoregressive (SVAR) models have become one of the primary tools to study macroeconomic dynamics. Macroeconomic variables are inherently endogenous, which complicates the application of traditional econometric methods to evaluate causal relationships. The SVAR framework allows to model endogenous interdependencies.

VARs are generally estimated in reduced form, i.e. without contemporaneous relationship between the endogenous variables in the system. While the reduced form summarizes the data, we are not able to interpret how the endogenous variables affect each other as the reduced form residuals are not orthogonal. The recovery of structural parameters and shocks requires identification restrictions that reduce the number of unknown parameters of the structural model.

A widely applied assumption on the contemporaneous relationships among the endogenous variables is the recursive ordering of the VAR, as this is straight-forward to implement with a

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Cholesky decomposition of the Variance-Covariance Matrix of the reduced form residuals. However, the recursive ordering is only plausible given a clear chain of causation. In a number of macroeconomic applications this is difficult to justify, in particular given the frequency of the data. The sign-restriction approach, in contrast, represents an identification scheme, which does not require to determine the sequence of causation in the model. In contrast, it allows that all variables respond to identified shocks simultaneously. The intuition of the sign restriction identification scheme is to consider all possible permutations of SVAR models corresponding to the reduced-form representation, but only to retain those that yield "economically reasonable" impulse responses. The sign-restriction approach dates back to Faust (1998), Canova and Nicolò (2002) and Uhlig (2005) and since then has been applied frequently to identify a broad set of macroeconomic shocks (Fry and Pagan, 2011). So far, however, ready-to-use selection algorithms that can easily be adjusted to a wide range of applications are scarce.[1]

In this vignette we describe a MATLAB routine, which allows researchers to estimate VARs in an Uhlig (1994) (Bayesian) fashion and identify shocks using either sign restrictions, zero restrictions or a combination of zero and sign restrictions. Sign and zero restrictions are imposed using selection matrices. Our algorithm is based on Arias et al. (2014). As we focus in this description on the application of our MATLAB routine, we ask the reader to refer to the original paper for technical details.

The routine offers the user a wide range of options that can be specified. This renders the routine extremely flexible and allows to replicate most algorithms put forward in the literature. While the routine offers a set of illustrations that pertain to percentiles of the distribution of the structural models, the core of the routine is the set of structural models in impulse response function (IRF), forecast error variance decomposition (FEVD) and historical decomposition (HD) representation. Using the set of structural models, the user can produce customized graphs and statistics, and use the structural models for further analysis. Using the set of identified models, the user may want to implement e.g. elasticity bounds (Kilian and Murphy, 2012), attain robust error bands (Giacomini and Kitagawa, 2015), or importance sample to reshape the distribution of structural models (Arias et al., 2018).

The description of ZeroSignVAR is structured as follows. In Section 2 we sketch first the idea of sign restrictions, and lay out an algorithm to implement them in Section 3. Section 4 describes the ZeroSignVAR package and Section 5 illustrates ZeroSignVAR using empirical data from the US.

[1] See Danne (2015) for a sign restriction algorithm implemented in R that allows to identify one single structural shock.

2 The Idea of Sign Restrictions

Consider a VAR(1) in structural form (without a constant term):

$$Y_t = B_0 Y_t + B_1 Y_{t-1} + \epsilon_t, \quad (1)$$

where Y_t represents the vector of n endogenous variables, B_0 captures the contemporaneous relationships, B_1 is the coefficient matrix at lag 1, and ϵ_t is a vector of white noise reduced form residuals with $\epsilon \sim N(0,1)$ and $\Sigma_\epsilon = E(\epsilon_t \epsilon_t') = I$. To estimate equation (1) we have to dispense with the contemporaneous endogenous variables Y_t on the right hand side:

$$\begin{aligned}(I - B_0)Y_t &= A_1 Y_{t-1} + \epsilon_t \\ B_0^* Y_t &= A_1 Y_{t-1} + \epsilon_t, \text{where } B_0^* = I - B_0 \\ Y_t &= B_0^{*-1} A_1 Y_{t-1} + B_0^{*-1}\epsilon_t.\end{aligned}$$

Now the VAR(1) can be estimated in reduced form:

$$Y_t = A_1 Y_{t-1} + u_t, \quad (2)$$

where the reduced form coefficients A_1 and u_t represent weighted averages of the structural coefficients B_1 and ϵ_t. In particular, $A_1 = B_0^{*-1} A_1$ and $u_t = B_0^{*-1}\epsilon_t$. While u_t is serially uncorrelated and exogenous, the variance-covariance matrix Σ_u is not diagonal and therefore the innovations u_t lack a structural interpretation.

To retrieve the structural parameters, we require additional information about the contemporaneous relationships B_0^*. Since, $\Sigma_\epsilon = I$, and $\Sigma_u = E(u_t u_t') = B_0^{*-1} E(\epsilon_t \epsilon_t') B_0^{*-1\prime} = B_0^{*-1} B_0^{*-1\prime}$, we require $n(n-1)/2$ restrictions to exactly identify the structural parameters B_0^*, B_1, and ϵ_t. The structural parameters are commonly identified using a Cholesky decomposition of the reduced form variance-covariance matrix Σ_u (as originally suggested by Sims, 1980), or specific short-run or long-run restrictions derived from theory (see e.g. Blanchard and Quah, 1989; Galí, 1992; Cecchetti and Karras, 1994).

To illustrate the idea of sign restrictions, let us write the reduced form VAR(1) in moving average representation:

$$Y_t = \sum_{i=0}^{\infty} \phi_i u_{t-i}, \quad (3)$$

where ϕ captures the reduced form impulse responses with $\phi_0 = I$ and $\phi_i = \sum_{j=1}^{i} \phi_{i-j} A_j$. In case of Cholesky, with $\Sigma_u = PP'$, we would obtain structural impulse responses $\psi_i = \phi_i P$ since with $Y_t = \sum_{i=0}^{\infty} \phi_i P P^{-1} u_{t-i}$, the structural variance-covariance matrix $\Sigma_\epsilon = P^{-1} E(u_t u_t') P^{-1\prime} = P^{-1}\Sigma_u P^{-1\prime} = P^{-1} P P' P^{-1\prime} = I$. While the Cholesky decomposition imposes a recursive structure in the VAR, in case of sign restriction we identify structural shocks by imposing restrictions on the signs of ψ_i over a specific horizon i. Hence as we do not exactly identify the impact matrix B_0^* as different orthogonalizations of the reduced form models are potentially consistent with the required sign restrictions. To obtain another orthogonal representation of the impulse responses in Equation (3), e.g. $\tilde{\psi}_i$, we can simply multiply ψ_i=$\phi_i P$ by a random matrix Q with the property $Q'Q = I$, since then it still holds that $\tilde{\Sigma}_\epsilon = E(Q'P^{-1}u_t u_t' P^{-1\prime} Q) = I$.

3 The Identification Algorithm

Our identification algorithm follows along the same steps:

1. Estimate the reduced form VAR.
2. Obtain orthogonal impulse responses by multiplying the reduced form responses ϕ_i with the lower triangular Cholesky factor P, from the decomposition of the reduced form variance-covarinace matrix $\Sigma_u = PP'$, and a random orthonormal matrix Q, which satisfies that $Q'Q = I$.
3. Check whether orthogonal impulse responses fulfill sign restrictions.
4. If yes, the orthogonal impulse responses bear a structural interpretation and are saved.
5. If not, repeat steps 2 and 3.

Step 1 In ZeroSignVAR the first step can be either estimated with ordinary least squares (OLS) or using a Bayesian approach. In the Bayesian approach we follow Uhlig (1994) and estimate the reduced form coefficients using an uninformative Normal-Inverse-Wishart prior, and obtain the posterior distribution, which is again a Normal-Wishart density, using the estimates $\hat{A}$ and $\hat{\Sigma_u}$ from an OLS regression as location parameters.

Step 2 In the second step we follow either Rubio-Ramírez et al. (2010) or Arias et al. (2018) depending on whether only sign restrictions or a combination of sign and zero restrictions are imposed. In case of pure sign restrictions, Rubio-Ramírez et al. (2010) show that it is an efficient way to obtain a random orthonormal matrix Q using a QR decomposition on a random matrix X (drawn from the normal distribution) with dimension $n \times n$, where n is the number of endogenous variables. In contrast, when zero restrictions are imposed, we follow Arias et al. (2018) and construct the matrix Q recursively. The recursive construction of Q allows to obtain an orthonormal matrix which also ensures that the transformed impulse responses ($\tilde{\psi}_i = \phi_i PQ$) are zero when required.

To improve the efficiency of the identification algorithm, we allow that the algorithm may rearrange the columns of Q if applicable. In case of pure sign restrictions, consider for instance that we identify aggregate demand, aggregate supply and monetary policy shocks plus one residual shock. Now given one specific draw of Q, it might be the case that $\tilde{\psi}_i$ fulfill the restrictions of the first (aggregate demand) shock but the responses of the second and third shocks are exactly interchanged, i.e. the responses of the second shock are consistent with a monetary policy shocks, while the responses of the third shock are consistent with an aggregate supply shock. As the impulse responses simply do not fulfill the imposed restriction due to the ordering of shocks, our algorithm rearranges the columns of Q such that the restrictions are fulfilled. More specifically, the algorithm always works through the columns of Q sequentially and compares each orthogonal shock (i.e. each column of Q, q_j) with all sets of impulse responses corresponding to the different structural shocks, starting always with the structural shock that has the highest number of sign restrictions. In case of zero restrictions, the algorithm compares only columns of Q with those structural shocks that have exactly the same zero restrictions.

While this sequential check of the sign restrictions substantially improves the efficiency of the algorithm, it also makes sure that no structural shock is falsely classified as residual shock. Note that although shocks are orthogonal by construction it still might be the case that two orthogonal shocks reveal the same structural properties given the imposed restrictions. In other words, it is principally possible that one specific Q captures two orthogonal shocks which produce impulse responses that are both consistent with the imposed restrictions of one structural shock. In other words, orthogonality is not a sufficient condition in case of sign restrictions to distinguish between structural shocks.

Given that a model is only partially identified, i.e. that some shocks are not identified, it therefore might be the case that orthogonal shocks, which bear a structural interpretation, are wrongly classified as residual shocks. While this should not be problematic for the analysis of impulse responses of the identified shocks, it might matter for variance decompositions. Therefore, our algorithm ensures that orthogonal shocks do not bear the same structural interpretation.

Step 3 To check the sign restrictions in the third step, we follow Rubio-Ramírez et al. (2010) and Arias et al. (2018) and specify for each shock j a selection matrix S which allows to check the imposed restriction with the simple condition that:

$$S_j f(\psi_h) q_j > 0, \text{for } h = 0, ..., H \text{ and } 1 \leq j \leq n, \tag{4}$$

where $f(\psi_h)$ is a vertically stacked matrix containing the Cholesky orthogonalized impulse responses ψ_h, H is the maximum horizon of the imposed sign restrictions, and q_j indicates the j^{th} column of Q. As S is a selection matrix, each row specifies exactly one sign restriction (1 in case of a positive response and -1 in case of a negative response). Therefore, the row dimension is determined by the number of sign restrictions s_j. The columns of S correspond to the rows of $f(\psi_h)$, with the dimension of $n(H+1)$ and thereby specify on which variable and what horizon h the sign restriction should be imposed.

Example To identify an aggregate demand shock let us assume that output, prices and the discount rate should all respond with the same sign on impact and the next period. The three variables are the only variables in our toy example. Furthermore, the aggregate demand shock is the first shock we identify. We impose in total 6 restrictions on responses of 3 variables, over a maximum horizon $H = 1$ and hence, S_1 has a dimension of 6×6. Given the order of the VAR is output, prices and the discount rate, S_1 is ordered as follows:

$$S_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Note that the rows of S_j can be ordered arbitrarily, but not the columns as they refer to the order of variables in the VAR. More specifically, the columns of S_1 correspond to the responses of output, prices and the discount rate in $h = 0$, and to the responses of output, prices and the discount rate in $h = 1$. Hence, the sign restriction in the first row specifies that output reacts positively to the AD shock on impact. The second row sets the restriction for the output response in the subsequent period. The third row refers to the sign restriction on prices on impact. Etc.

Zero restrictions, in contrast to sign restrictions, hold by construction. In case zero restrictions are specified, the orthogonal matrix Q is obtained recursively taking into account the imposed zero restrictions. Due to the recursive construction of Q, shocks have to be ordered according to the number of zero restrictions, starting with the shock with the highest number of zero restrictions. One has to specify first the shock with the highest number of zero restrictions.

Arias et al. (2018) show that zero restrictions are linear restrictions on each column of the orthogonal matrix Q. More specifically, the j column of Q is given by

$$q_j = \frac{k_{j-1}k'_{j-1}x_j}{\left\|k_{j-1}k'_{j-1}x_j\right\|}$$

for $1 \leq j \leq n$ and any vector k_{j-1} whose column form an orthonormal basis for the null space of the $(z_j + \max\{1, j-1\}) \times n$ matrix $R_j = [(Z_j f(\psi_h))' \; q_1 ... q_{j-1}]'$, where $\|\|$ is the Euclidean norm, z_j is the number of zero restrictions for each shock j, and x_j is the j^{th} column of a $n \times n$ matrix X that contains normally distributed random numbers.[2]

Example Consider again a small monetary VAR, including an output measure, prices and the discount rate. To identify a monetary policy shocks we might assume that changes in output

[2] In MATLAB we define `K = null(R).`

and prices influence monetary policy contemporaneously but that policy measures affect the economy only with a one period lag (see e.g. Christiano et al., 1999). Given that the policy shock is the only shock we want to identify with zero restrictions (and hence it is the shock with the highest number of zero restrictions) it has to be the first shock that we identify. Furthermore consider that the order of the variables is output, prices and the policy measure. As we impose zero restrictions on two variables on impact Z_1 has a dimension of 2×2, where the ones are ordered as follows:

$$Z_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Step 4 In the fourth step, given the imposed sign restrictions are fulfilled (zero restrictions always hold by construction), the structural impulse responses are saved. An option specifies, whether the algorithm proceeds with Step 2 and checks another Q-transformations, until a specific number of Q-draws has been checked, or in case of the Bayesian estimation, proceeds with Step 1, and draws another model from the reduced form posterior distribution, until a specific number of reduced form model has been drawn.

Step 5 When the imposed sign restrictions are not fulfilled, the algorithm proceeds with Step 2 and 3 and checks a new draw of matrix Q. When the maximum of Q transformation set in the options is reached and no Q matrix has fulfilled the restrictions, the algorithm proceeds with another model draw in case of the Bayesian estimation (as described in Step 4) or ends with the error message `'No model fitting the restrictions was found.'` in case of the OLS estimation. The same error message is displayed if no model is found for all draws from the reduced form posterior distribution in case of the Bayesian approach.

4 The ZeroSignVAR Package

To use ZeroSignVAR, download the zip-file ZeroSignVAR.zip from one of the authors' homepages and extract its content. The folder contains a clean startZeroSign.m file, a folder called functions, this vignette in pdf format, two example files (Model_01.m, Model_02.m) file, and the corresponding data.xls file. The code is organized such that only the startZeroSign.m file has to be modified for the personal application. In this file the user specifies the available options, the data and the sign and zero restrictions. When the start file is specified, the user simply hits the run button and all results are saved in a specified folder. The example files apply the ZeroSignVAR algorithm to U.S. data as described in Section 5.

4.1 Options

Using the structure environment called `opt` the user may adjust several options. The structure environment allows to easily add new options if applicable for the individual use of ZeroSignVAR. While several options are set to default values when missing, the mandatory options have to be specified by the user. When a mandatory option is missing the algorithm stops with a build in error message. The options include general program settings, VAR and data specifications, options for the estimation and identification, and results and plot options. In the following we describe each option and indicate whether it is a mandatory option [M] and whether the option has to be specified in text [STR, LIST] or numerical format [SCA, VEC]. STR indicates a single string element (`'String Element'`), LIST is a list of string elements (`{'String ... Element 1', 'String Element 2'}`), SCA indicates a scalar value, and VEC is a sleeping vector (`[1, 2, 3]`). The options can be specified in any order:

`opt.modelName` Choose a unique model name. A folder will be created with this name in which all result files are saved. E.g. `'Model_01'` [M, STR]

`opt.modelPath` Specify the path to the folder, in which all result files should be saved. [M, STR]

`opt.lVars` Define labels for the variables in the same order as they enter the VAR, i.e. according to the columns of the data matrix y. E.g. `{'GDP', 'CPI', 'FFR'}`. [M, LIST]

`opt.lShocks` Define labels for the identified shocks. The number of shock labels has to be equal to the number of variables (i.e. label also residual shocks). [M, LIST]

`opt.startDate` Define the start date of the data (MM-DD-YYYY). [M, STR]

`opt.endDate` Define end date (MM-DD-YYYY). [M, STR]

`opt.nLags` Define the lag order. [M, SCA]

`opt.estimationMethod` Choose the estimation method. Available options: `'OLS'` or `'diffuse'`, corresponding to either ordinary least square or Bayesian estimations. [M, STR]

`opt.runNumber` Specify the number of a run. This option can be used to add additional iterations to an existing estimation. The run number controls that different random numbers are used. The default value is 1. If you want to perform more runs, the number of runs has to be increased in consecutive steps. [SCA]

`opt.hasConstant` USe a vector of constants in the estimation of the VAR (1 = Yes, 0 = No; Default = 1). [SCA]

`opt.hasTrend` Use a linear trend in the estimation of the VAR (1 = Yes, 0 = No; Default = 0). [SCA]

`opt.nMaxSignHorizon` Maximum horizon of sign restrictions (0 = only contemporary, 1 = contemporary + horizon 1, etc.). [M, SCA]

`opt.nDrawsFromBvar` Number of draws to generate posterior distribution for BVAR. This is only relevant when the estimation method is non-analytical, i.e. set to `'diffuse'` (Default = 1000). [SCA]

`opt.nTransformationsPerDraw` Number of Householder transformations per model draw (Q-transformation; Default = 1000). [SCA]

`opt.drawFromPosterior` Draw from posterior distribution when working through sign restrictions algorithm (1 = Yes, 0 = work through all draws; Default = 0). [SCA]

`opt.nModelDraws` Number of model draws. This is only used if `opt.drawFromPosterior` == 1. [SCA]

`opt.checkNegativeQ` Check also if sign restrictions are fulfilled for the negative of a Q column and transform this column of Q to it's negative if applicable. If this option is 0 the diagonal of Q will be normalized to have a positive diagonal (Default = 1). [SCA]

`opt.keepAllValid` Keeps all valid transformations when transforming one particular draw from the posterior (1 = Yes, 0 = only transform until you find a match; Default = 0). [SCA]

`opt.nMatches` Stop search-loop, when a certain number of models fulfill restrictions (Default = 1,000,000). [SCA]

`opt.nImpulseHorizon` Define the horizon over which impulse responses are calculated (0 = only contemporary, 1 = contemporary + horizon 1, etc., Default = 12). [SCA]

`opt.isIRFcum` Indicate if a variables should be cumulated (Default =0). Note that sign restrictions are imposed on the cumulative responses accordingly. E.g. Vars 3 and 4 should be cumulated: `opt.isIRFcum = [3,4];` [VEC]

`opt.nCTMHorizon` Define the horizon over which the closest-to-median impulse response is calculated (0 = only contemporary, 1 = contemporary + horizon 1, etc.). The Default is set to the horizon of the impulse response functions. The closest-to-median model is selected as suggested in Fry and Pagan (2011). [SCA]

`opt.narrowCTMSearch` When looking for the closest-to-median impulse response, use only impulse responses associated with structural shocks (1 = Yes, 0 = No, use all impulse responses; Default = 0). [SCA]

`opt.isNoTransform` Debug mode. The algorithm does not work through the zero- and sign restrictions (for debugging; 1 = Yes, 0 = No; Default = 0). Shocks are orthogonalized using only the Cholesky decomposition. [SCA]

`opt.isSaveResults` Save identified models in results.mat (1 = Yes, 0 = No; Default = 0). [SCA]

`opt.isIRFtable` Calculate impulse response functions for all identified models and save closest-to-median, point-wise median and percentiles in irf.mat (1 = Yes, 0 = No; Default = 0). [SCA]

`opt.isStructShockTable` Calculate structural shocks for all identified models and save closest-to-median, point-wise median and percentiles in structShocks.mat (1 = Yes, 0 = No; Default = 0). [SCA]

opt.isFevdTable Calculate the FEVD for all identified models and save closest-to-median, point-wise median and percentiles in fevd.mat (1 = Yes, 0 = No; Default = 0). Note this slows down the program. [SCA]

opt.isFevdTexTable Save point-wise median and one standard deviation percentiles in a LateX-Table (1 = Yes, 0 = No; Default = 0). Note this option requires the calculation of the FEVD for all identified models and thereby slows down the program. [SCA]

opt.isHistDecompTable Calculate the historical decomposition for all models and save the closest-to-median, the point-wise median, and percentiles in HistDecompCTM.mat and HistDecompPWM.mat, respectively (1 = Yes, 0 = No; Default = 0). Note this slows down the program. [SCA]

opt.isImpulsePlots Generate impulse response plots (1 = Yes, 0 = No; Default = 1; point-wise median values). [SCA]

opt.isPlotTitle Print labels above impulse responses (1 = Yes, 0 = No; Default = 0) [SCA]

opt.isPlotCTM Plot closest-to-median responses (1 = Yes, 0 = No; Default = 0). [SCA]

opt.nPlotRandomModels Choose number of randomly drawn models to include in the impulse response plots (0 = none; Default = 0). [SCA]

opt.isFevdPlots Generate plots of the FEVD (1 = Yes, 0 = No; Default = 0; point-wise median values). [SCA]

opt.isStructShockPlots Generate plots of structural shocks (1 = Yes, 0 = No; Default = 0; point-wise median values). [SCA]

opt.isHistDecompPlots Generate plots of the historical decomposition (1 = Yes, 0 = No; Default = 0; point-wise median values). [SCA]

4.2 Data Input

MATLAB provides several ways to import data from external sources. Please refer to the MATLAB help files for this purpose. The data should be finally contained in a matrix labeled y, in which the variables are ordered along the column-dimension and observations (periods) across the row-dimension.

4.3 Imposing the restrictions

The sign and zero restrictions are implemented using selection matrices S and Z. S and Z are three dimensional arrays (i.e. stacked matrices). Dimensions one and two (i.e. rows and columns) are described above. The third dimension refers to the number of the shock that is identified.

Examples

1. First shock: zero restriction on variables 1 and 2 at horizon 0

```
Z(1,1,1) = 1;
Z(2,2,1) = 1;
```

2. Second shock: negative reaction of variable 1 up to maximum horizon

```
for ii = 1:(opt.nMaxSignHorizon+1)
    S(ii,1+nVars*(ii-1),2) = -1;
end
```

3. Second shock: positive reaction of variable 4 at horizons 0 and 1

```
S(opt.nMaxSignHorizon+2,4,2) = 1;
S(opt.nMaxSignHorizon+3,4+nVars,2) = 1;
% Note: As we already have nMaxSignHorizon+1 sign restrictions for the second
% shock we have to start at nMaxSignHorizon+2.
```

4. For shocks without restrictions (residual shocks) nothing has to be done, but should be normalized with a sign restriction on the impact response of one variable (Giacomini and Kitagawa, 2015).

5 A Simple Application

In this section we describe a small application and all necessary steps to obtain the results saved in the folder "SimpleApplicationResults". Specifically, we evaluate the effects of a (i) sign-identified and (ii) zero- and sign-identified monetary policy shock on output. Furthermore, to cross-validate our sign-restriction algorithm, we use the same data and restrictions as in Moon et al. (forthcoming) and Giacomini and Kitagawa (2015). Table 2 summarizes the two different identification approaches.

Table 2: Identification schemes: Contractionary monetary policy shocks

Identification	Shock	RGDP	INF	FFR	MONEY
Sign-Restrictions	MP Shock		≤ 0	≥ 0	≤ 0
Zero- and Sign-Restrictions	MP Shock	0	≤ 0	≥ 0	≤ 0

Notes: The empty entry indicates that no restriction is imposed. Sign restriction hold on impact and the next period. Zero restrictions hold only on impact. All residual shocks are normalized with a positive response on impact of the respective variable.

We start with the identification approach used in Moon et al. (forthcoming) and Giacomini and Kitagawa (2015)—the first row in Table 2. Therefore, we estimate a VAR including data on real GDP per capita, inflation, the federal funds rate and the real money balance. We follow the description in the online technical appendix of Moon et al. (forthcoming) and obtain all data from the St. Louis Federal Reserve Bank database (FRED), except the data on money supply, which is taken from Cynamon et al. (2006).[3] We apply exactly the same data transformation as suggested in Moon et al. (forthcoming). Table 3 summarizes the dataset.

The VAR with two lags is specified as follows:

$$Y_t = c + \sum_{j=1}^{2} A_j Y_{t-j} + u_t, \tag{5}$$

where Y_t is the vector of endogenous variables, c is a constant term, A_j is the matrix of reduced form coefficients at lag j, and e_t is a vector of white noise residuals with $e_t \sim (0, \Sigma_e)$.

Table 3: Data

Variable	Definition (Data-Codes)
RGDP	Real GDP per capita is calculated by first dividing real GDP (GDPC96) by the non-institutionalized population series (CNP16OV; using quarterly averages). The natural log of real GDP per capita is linearly de-trended using an OLS regression over the period from 1959Q1 to 2006Q4. The cyclical component is finally multiplied by 100 to convert the deviation to percentages.
INF	The inflation rate is calculated using log differences of the GDP deflator (GDPDEF). The variable is scaled by 400 to yield an annualized rate.
FFR	The nominal interest rate is measured with the quarterly averages of the effective Federal Funds rate (FEDFUNDS)
MONEY	Real money balance is defined as the quarterly averages of the sweep-adjusted M2 money stock (M2S), provided by Cynamon et al. (2006), divided by the GDP deflator (GDPDEF). In contrast to Moon et al. (forthcoming), we do not seasonally adjust the M2S series, as the data is already available as seasonally adjusted series. From the natural log of real money balance, a linear trend is extracted (using again OLS over the period from 1959Q1 to 2006Q4) and the cyclical component is multiplied by 100.

Notes: All data are obtained from the St. Louise Federal Reserve Bank database (FRED), except the sweep-adjusted M2 money stock. The final dataset is restricted to the period from 1965Q1 to 2005Q1. The variable definitions follow exactly the instruction from Moon et al. (forthcoming), in order to obtain the same dataset.

First we have to specify all mandatory options (e.g. use the plain `StartZeroSignVAR.m` file):

```
opt.modelName = 'SimpleApplicationResults';
opt.modelPath = pwd;
opt.lVars = {'FFR', 'GDP', 'INF', 'M2'};
opt.lShocks = {'MP Shock', 'Shock2', 'Shock3', 'Shock4'};
opt.startDate = '01-01-1965';
opt.endDate = '01-01-2005';
opt.nLags = 2;
opt.estimationMethod = 'diffuse';
```

[3] The data is available at `http://www.sweepmeasures.com`.

We set `opt.modelPath = pwd`, such that the present working directory of MATLAB is used as the destination directory for the results. In addition we set the sign restriction horizon to 1.

```
opt.nMaxSignHorizon = 1;
```

Then we have to load the data as prepared in Table 3, and specify matrix y:

```
y = xlsread('data.xls','A1:D161');
```

Only Sign Restrictions To impose the sign restrictions we have to specify the S matrix:

```
for ii = 1:(opt.nMaxSignHorizon+1)
    % Positive response of FFR
    S(ii,1+nVars*(ii-1),1) = 1;
    % Negative response of inflation
    S(ii+opt.nMaxSignHorizon+1,3+nVars*(ii-1),1) = -1;
    % Negative response of M2
    S(ii+2*(opt.nMaxSignHorizon+1),4+nVars*(ii-1),1) = -1;
end

% Normalizations of residual shocks
S(1,2,2)=1;
S(1,3,3)=1;
S(1,4,4)=1;
```

Zero- and Sign Restrictions Additionally to the sign restrictions we now also impose the zero restriction that output (RGDP) does not respond contemporaneously to the monetary policy shock.

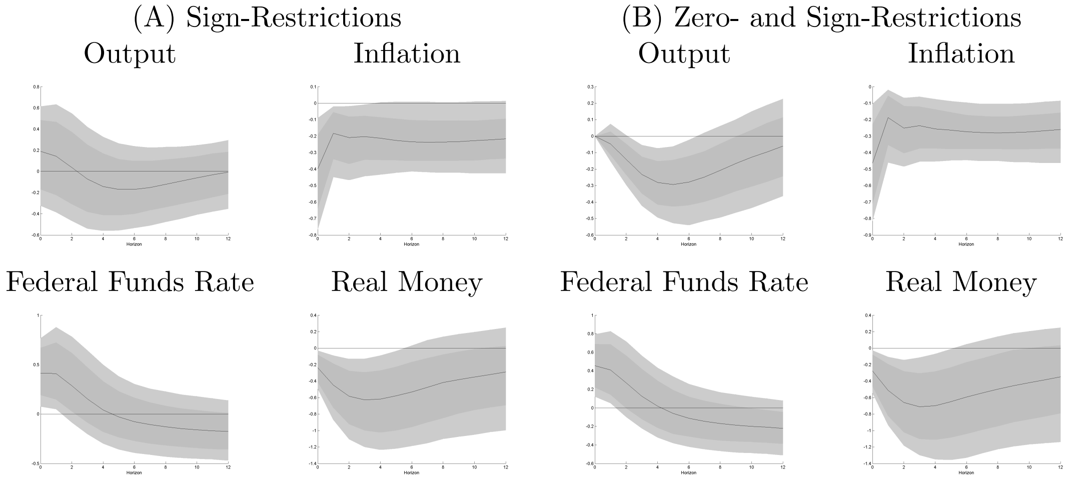
```
for ii = 1:(opt.nMaxSignHorizon+1)
    % Positive response of FFR
    S(ii,1+nVars*(ii-1),1) = 1;
    % Negative response of inflation
    S(ii+opt.nMaxSignHorizon+1,3+nVars*(ii-1),1) = -1;
    % Negative response of M2
    S(ii+2*(opt.nMaxSignHorizon+1),4+nVars*(ii-1),1) = -1;
end

Z(1,2,1) = 1;

% Normalizations of residual shocks
S(1,2,2)=1;
S(1,3,3)=1;
S(1,4,4)=1;
```

Figure 1 shows the impulse responses that are saved in the respective directory. Panel (A) replicates the Bayesian error bands shown in Figure 2 of Moon et al. (2013, p. 33) and the output response in Panel (B) replicates the Bayesian error bands of the output response shown in Figure 1 (Model III) of Giacomini and Kitagawa (2015, p. 32).

Figure 1: Impulse responses to monetary policy shocks



Notes: The light gray and dark gray areas represent 90% and two third of the identified posterior distribution.

6 General Info

If you apply this routine or use substantial parts of the code please cite this unpublished manuscript as:

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Please let us know if you find any mistakes. We will provide updates on our personal web pages:
`http://eeecon.uibk.ac.at/~breitenlechner`
`http://eeecon.uibk.ac.at/~geiger`

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