Bonus-vetus-OLS for Gravity Models and Its Approximation Error

Revised Version

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Abstract

The use of high-dimensional fixed-effects estimation has become customary with the estimation of gravity models of bilateral trade, migration, or commuting as outcome. However, fixed-effects methods can be used without incidental-parameter bias in a very small set of stochastic models. Alternatives to fixed-effects estimation are iterative-structural model estimation or linearizations of the structural model. Baier and Bergstrand (2009a) deployed such a linearization. While easy to implement, the approach has drawbacks related to the approximation point and lack of observability of ingredients needed for the linearization. This compromises empirical work. The present paper provides a remedy to this problem by linearizing at the observed trade equilibrium.

Keywords: Bilateral trade flows; Gravity equation; OLS estimation; Linearization
JEL codes: C23; F12; F17

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1 Introduction

Since the seminal contributions of Eaton and Kortum (2002) and Anderson and van Wincoop (2003), the treatment of producer-country and customer-country price indices as endogenous variables has become important in structural-quantitative work in trade. Anderson and van Wincoop (2003) proposed a well-known iterative approach to the problem, which is nonlinear in the structural trade-cost parameters. Rather than pursuing such iterative-structural estimation, it is now customary to control for country-(time-)specific endogenous terms by including high-dimensional fixed effects. The latter usually include at least exporter-time- and importer-time- and sometimes additionally exporter-importer-fixed effects.

The structural-iterative or high-dimensional-fixed effects approaches had been used originally in (log-)linear stochastic outcome-equation models. There, the right-hand-side index of the bilateral-trade outcome equation is – conditional on exporter-time and importer-time price and income terms or on respective fixed effects – log-linear in the parameters of observable trade-cost variables such as geographical distance, ad-valorem trade-cost factors, etc. Since the publication of Santos Silva and Tenreyro (2006) it became customary to avoid potential parameter biases of log-linear gravity models by using Poisson pseudo-maximum-likelihood (PPML) estimation instead. For the latter, Weidner and Zylkin (2021) established properties and solutions when using high-dimensional fixed effects.

While most applications of gravity models use either (log-)linear regressions or Poisson pseudo-maximum-likelihood estimation as two special forms of so-called generalized linear models in conjunction with high-dimensional fixed effects, there may be reasons for not doing so. In a nutshell, these are the following. First, a two-way or higher-way fixed-effects model may entail an over-parameterization of the structural model. The importer-time fixed effects are a nonlinear function of the exporter-time fixed effects and of trade costs only (see Eaton and Kortum, 2002, or Anderson and van Wincoop, 2003). The number of unknown parameters in the trade-cost function is typically much smaller than the number of importer-time fixed effects. Hence, when not imposing structural constraints on the importer-time fixed effects, one wastes degrees of freedom and estimates too many parameters relative to the structural model. Employing importer-time-fixed effects may, hence, lead to upward-biased confidence intervals around the structural parameters of interest.

Second, tests might reject the distributional assumptions imposed by log-linear OLS or by PPML (see Egger and Staub, 2013) with consequences for consistency of the high-dimensional fixed-effects design. Special forms of this case include ones where the outcome is binary rather than continuous, e.g., with an entry-selection equation that otherwise adheres to the gravity form (see Helpman, Melitz, and Rubinstein, 2008). Another case is a fractional model, where bilateral export or import shares rather than levels are used (see Papke and Wooldridge, 2008, for a framework but an application in a non-trade context). Yet another case are nonparametric models (see Gallo, Marzano, and Simonelli, 2012; Carrère, Mrázová, and Neary, 2020). Such situations do not permit the estimation with high-dimensional fixed effects in a straightforward way. Clearly, these and others are contexts, where structural estimation or an approximation of the fixed effects through linearization of the structural model are desirable.

Third, the identification of parameters relating to unilateral determinants of bilateral flows has to separate their impact from that of multilateral resistances which exhibit unilateral variation only as well. These unilateral variables relate to non-discriminatory trade policies, exporter specific R&D-expenditures, productivity, institutions or the quality of government. With structural-iterative or linearized-model estimation, the impact of these unilateral variables on bilateral trade flows can then estimated along with the multilateral resistance terms (see e.g., Francois and Manchin, 2013; Márquez-Ramos, 2016; Bratt, 2017; Sellner, 2019; Barbero, Mandras, Rodríguez-Crespo and Andrés Rodríguez-Pose, 2021). High-dimensional fixed-effects estimators, either based on log-linear OLS or on PPML, are incapable of identifying the parameters of these unilateral variables.

Fourth, iteratively solved or linearized multilateral resistance terms are sometimes used to construct instruments of trade flows in regressions of outcomes that depend on trade (see, e.g., Felbermayr and Gröschl, 2013, Clougherty and Grajek, 2014). Clearly, these instruments could not be simply replaced by fixed country effects in such contexts.

In a widely-cited contribution, Baier and Bergstrand (2009a) proposed a linearization of the structural outcome equation of bilateral trade flows as an elegant way to avoid (i) the structural-iterative estimation of the model parameters or (ii) the use of high-dimensional fixed effects. Unlike structural-iterative estimation, this approach can rely on linear one-shot regressions, and unlike high-dimensional fixed-effects estimation, it does not inflate the number of parameters to be estimated, it provides parameters on country- or country-time-specific regressors, and it can be used in nonlinear and even nonparametric estimation. When taking logs of the model's right-hand side – and this is done both with log-linear OLS as well as with PPML – bilateral trade flows can be modelled as to be additive in observable trade-cost measures (log distance, binary indicators for common language, history, etc., and log ad-valorem tariff factors) times parameters and in so-called multilateral resistance terms (see Anderson and van Wincoop, 2003), which depend nonlinearly on trade costs and their parameters. To retain a (log-)linear estimation framework, Baier and Bergstrand (2009a) linearize the system of multilateral resistance terms as a function of trade costs and their parameters. This linearization depends on the sales and expenditure shares of all countries.

Due to the isomorphic structure of a host of alternative quantitative trade models, as outlined in Arkolakis, Costinot, and Rodríguez-Clare (2012), the iterative nonlinear approach is and its linear approximation should be applicable for estimating the key parameters as well as for enabling counterfactual equilibrium quantifications of a class of customary, quantitative models of bilateral international trade. Not surprisingly, the proposed linearization enjoys a high popularity because of its simplicity. As of March 10 in 2024, Baier and Bergstrand (2009a) received 940 cites, according to Google Scholar, and the approach is impactful not only in international economics focused on trade (see Carrère, 2006; Baier and Bergstrand, 2009b; Egger and Nelson, 2011; Nicita and Hoekman, 2011; Behar and Nelson, 2014; Van der Veer, 2016; Saia, 2017; Afesorgbor, 2019; Atalay, Hortaçsu, Li, and Syverson, 2019; Atif, Mahmood, Liu, and Mao, 2019; Baier and Standaert, 2020; Doe Fiankor, Flachsbarth, and Brümmer, 2020) or migration (Gröschl and Steinwachs, 2020), but also in environmental economics (see Felbermayr and Gröschl, 2013; Aichele and Felbermayr, 2015), in development and institutional economics (see Glick and Taylor, 2010; Berger, Easterly, Nunn, and Satyanath, 2013), in transport economics (see Behar, Manners, and Nelson; 2013; Zhang and Zhang, 2016), in business economics (Blind, Mangelsdorf, and Pohlisch, 2018), in political science (see Dür, Baccini, and Elsig, 2014), and even in demography (see Czaika and Parsons, 2017).

The present paper shows that the Bonus-vetus-OLS (BvOLS) linearization of Baier and Bergstrand (2009a) leads to a systematic approximation bias. The reason is that the sales and expenditure shares of all countries are not observable at the approximation point used for the linearization, where trade costs are zero. Due to the latter, Baier and Bergstrand (2009a) propose using and do use sales and expenditure shares as observed. The same is true for all applications of this procedures as cited above. But those shares do not correspond to the free-trade approximation point. As a result the approximation is biased and it does not even approximate the nonlinear structural trade model in the approximation point, where the linearization should not have any error.

As a general rule, linear, one-dimensional approximations of nonlinear functions involve three bits of information: first, the specification of the full system of equations to be approximated (with gravity models: the gravity equation and the balance-of-payments restrictions); second, the magnitude of the difference in the variable in whose dimension the system is approximated between the approximation point and the evaluation point (with gravity models: trade costs); third, the benchmark equilibrium values in which the system is approximated (i.e., all exogenous and endogenous variables; with country-pair-level gravity models: sales and expenditures, prices, and trade costs). Only if all three pieces of information are used, the linearized model will be tangential to the nonlinear one in the approximation point.

However, with BvOLS, only the first two pieces of information are used, but observed sales and expenditure shares (or GDP shares) are used instead of the required (unobserved) ones pertaining to the approximation point. This leads to two problems: first, the approximation error of the model associated with its linearization is exacerbated for non-zero trade costs; and, second, the linearized model does not approximate the nonlinear one even in the approximation point. Hence, BvOLS suffers from two sources of error: the standard linearization error and the lack of tangentiality of the linearized gravity equation in the approximation point.[1]

The present paper provides an alternative linearization of structural gravity models at the observed trade equilibrium rather than the unobserved free-trade equilibrium. We demonstrate that this approach avoids the fundamental approximation error of BvOLS while retaining the simplicity of application and lacking the limitation of being usable only with log-linear or PPML estimation for the estimation of parameters and their standard errors. Moreover, this model can even be used to compute approximated general-equilibrium responses of outcome.

While the general issues addressed above are clearly relevant to the international trade economist, they matter similarly for regional modellers. Over the past decades, numerous examples exist for researchers addressing subnational goods-trade flows between regions along the lines of gravity models (see Polenske, 1963; Black, 1972; Smith, 1987; Anderson and van Wincoop, 2003; Nitsch and Wolf, 2013; Cai, 2023; Egger, Loumeau, and Loumeau, 2023; or Azorín, Martínez Alpañez, and Sánchez de la Vega, 2024; this list is by no means exhaustive). However, the relevance goes beyond trade. E.g., there are approaches that address interregional migration flows with a similar structure, including the examples in Peeters (2012) or Egger, Loumeau, and Loumeau (2023), and ones for interregional commuting as in Ahlfeldt (2011) or Monte, Redding, and Rossi-Hansberg (2019). Importantly, the linearization provided here works for all these problems with the only difference being that the sales- and expenditure-share variables in trade-flow

[1] Baier and Bergstrand (2010) propose a variant of their linearization approach. However, the associated approximation point is one, where not only all (foreign and domestic) bilateral trade costs are symmetric but also all countries' endowments (and, hence, prices and GDPs) are symmetric. The approximation points in Baier and Bergstrand (2009a, 2010) are very distant from real-world data. Therefore, neither approach in Baier and Bergstrand (2009a, 2010) is well suited to predict the trade consequences of even small discrete changes on trade flows. However, while the customary approach in Baier and Bergstrand (2009a) suffers from a bias in the trade-cost-parameter estimates, this bias is absent in the approach of Baier and Bergstrand (2010). Moreover, the variant in Baier and Bergstrand (2010) shares the property with the high-dimensional fixed-effects estimator that parameters on country- or country-time-specific trade-cost variables cannot be identified.

gravity models would have to be replaced by other ratios suggested by economic theory (those would be resident ratios or resident and worker ratios relative to all regions, respectively).

In the subsequent section, we outline the model preliminaries underlying the BvOLS linearization. We introduce a correct linearization of the model and compare it with the BvOLS version. Section 3 addresses the estimation bias with BvOLS and envisages the quantification of counterfactual trade-shock responses. Section 4 provides an illustration of various biases addressed in earlier sections using real-world trade data. The last section concludes with a short summary of the main findings.

2 The Nonlinear System and Its Linearizations

2.1 The Gravity Model

The class of trade models of interest here are ones, where deterministic aggregate bilateral exports from country i to j, X_{ij}, can be formulated as:[2]

$$\widetilde{X}_{ij} \equiv X_{ij}/Y = e^{\alpha t_{ij}} \kappa_i \Pi_i^{-\alpha} \theta_j P_j^{-\alpha}. \tag{1}$$

Trade flows are normalized by world aggregate sales, $Y = \sum_{i=1}^{N} Y_i = \sum_{j=1}^{N} E_j$, with Y_i and E_j being country-level aggregate sales and expenditure value, respectively. $\kappa_i = \frac{Y_i}{Y}$ denotes the sales share of country i in the world, and $\theta_j = \frac{E_j}{Y}$ is the expenditure share of country j, respectively. $\alpha < 0$ is often referred to as the *trade elasticity*.[3] This formulation is more general than the one considered by Baier and Bergstrand (2009a), since it allows for country-specific trade imbalances. However, this will be immaterial to the key arguments below. The terms on the right-hand side of equation (1) measure the exponentiated direct effect of log bilateral, potentially asymmetric ad-valorem trade costs on exports from i to j, t_{ij}, the sales and expenditure shares introduced above, κ_i and θ_j, and the exporter and importer price indices or multilateral trade resistance terms, $\Pi_i^{-\alpha}$ and $P_j^{-\alpha}$, respectively, as introduced by Anderson and van Wincoop (2003). In a world with N countries, the

[2]Arkolakis, Costinot, and Rodríguez-Clare (2012) show that not only endowment-economy models as the one of Anderson and van Wincoop (2003), but also other ones such as Eaton and Kortum (2002) type Ricardian models, Krugman (1980) type models, or Chaney's (2008) parametrizations of Melitz (2003) type models can be represented in this way.

[3]In Armington models à la Anderson and van Wincoop (2003) or Dixit-Stiglitz-Krugman models with monopolistically competitive firms $\alpha = 1 - \sigma$ and σ denotes the elasticity of substitution between varieties. In Eaton and Kortum (2002) type models $\alpha = -\theta$, where θ measures the dispersion of productivity among perfectly competitive suppliers. In any case, α may be referred to as the partial effect (or the direct elasticity) of normalized bilateral trade flows (exports or imports) with respect to ad-valorem trade costs.

latter are defined – through utility maximization and multilateral market-clearing conditions – as

$$P_j^\alpha = \sum_{i=1}^{N} e^{\alpha t_{ij}} \kappa_i \Pi_i^{-\alpha}, \quad \Pi_i^\alpha = \sum_{j=1}^{N} e^{\alpha t_{ij}} \theta_j P_j^{-\alpha}, \quad i, j = 1, ..., N.$$

Trade frictions in logs are collected in the $N^2 \times 1$ vector $\mathbf{t} = (t_{ij})$ with corresponding parameter α.[4] The $N^2 \times 1$ vector $\widetilde{\mathbf{x}}(\mathbf{t})$ has typical elements $\ln(\widetilde{X}_{ij})$ and denotes world-sales-normalized bilateral exports in logs. Sales and expenditure shares are collected in the $N \times 1$ vector $\boldsymbol{\kappa}(\mathbf{t})$ and the $(N-1) \times 1$ vector $\boldsymbol{\theta}(\mathbf{t})$ with typical elements $\kappa_i \equiv \kappa_i(\mathbf{t})$ and $\theta_j \equiv \theta_j(\mathbf{t})$, respectively. This notation emphasizes that both depend on the trade frictions and, since trade may be unbalanced, they may differ from each other. For the derivations below, it will be useful to define $\mu_i(\mathbf{t}) \equiv \ln(\kappa_i(\mathbf{t})\Pi_i(\mathbf{t})^{-\alpha})$ and $m_j(\mathbf{t}) \equiv \ln(\theta_j(\mathbf{t})P_j(\mathbf{t})^{-\alpha})$, capturing the unilateral exporter and importer country-specific elements of the gravity model.

In an economy with endowment A_i of country i and in the absence of any country-specific Armington-type preference bias, sales shares obey

$$\kappa_i(\mathbf{t}) = \frac{e^{\frac{1}{\alpha}\mu_i(\mathbf{t})} A_i}{\sum_{k=1}^{N} e^{\frac{1}{\alpha}\mu_k(\mathbf{t})} A_k}. \tag{2}$$

There, k defines the running index of countries in the sum entering the denominator. This result uses the fact that the mill (or factory-gate) price in equilibrium can be written as $p_i = \mu_i^{1/\alpha}$. Moreover, world-sales-normalized exports can be expressed as

$$\widetilde{X}_{ij}(\mathbf{t}) \equiv \frac{X_{ij}(\mathbf{t})}{Y} = \exp(\alpha t_{ij} + \mu_i(\mathbf{t}) + m_j(\mathbf{t})).$$

Upon normalizing $m_1(\mathbf{t}) = 0$, bilateral trade flows adhere to the implicit-function system of multilateral resistances as introduced by Anderson and van Wincoop (2003):

$$\begin{bmatrix} \sum_{i=1}^{N} e^{\alpha t_{ij} + \mu_i(\mathbf{t}) + m_j(\mathbf{t})} - \theta_j(\mathbf{t}), \; j = 2, ..., N \\ \cdots \\ \sum_{j=1}^{N} e^{\alpha t_{ij} + \mu_i(\mathbf{t}) + m_j(\mathbf{t})} - \kappa_i(\mathbf{t}), \; i = 1, ..., N \end{bmatrix} = \mathbf{0}. \tag{3}$$

The system of trade-resistance terms comprises $2N - 1$ interdependent variables ($\mu_i(\mathbf{t})$ and $m_j(\mathbf{t})$) and as many equations.

[4] We concentrate on a model where t_{ij} is parameterized by a single explanatory variable to simplify notation. However, introducing several explanatory variables behind t_{ij} is straightforward. Then, we could think of the trade-cost function in logs to consist of K elements as $\alpha t_{ij} = \sum_{k=1}^{K} \zeta_k z_{k,ij}$. The parameter the researcher would estimate on observable trade-cost measure $z_{k,ij}$ is $\zeta_k = \alpha \delta_k$ (see Anderson and van Wincoop, 2003, and Pfaffermayr, 2020). In the latter, the parameter δ_k translates observable trade-cost measures into log ad-valorem equivalents.

2.2 Linearizing the Generic Structural Gravity Model

For the linearization of the above system we need to determine the point at which to linearize. We use subscript a to refer to this approximation point. Below we will consider several different approximation points with the generic subscript a taking on different values. As will become clear, this is of utmost importance here in order to not confuse variables and parameters relating to different approximation points. We use a to index all variables which are specific to the approximation point, namely the vector of trade costs $\mathbf{t}_a$, normalized exports $\widetilde{\mathbf{x}}_a \equiv \widetilde{\mathbf{x}}(\mathbf{t}_a)$, sales and expenditure shares $\boldsymbol{\kappa}_a \equiv \boldsymbol{\kappa}(\mathbf{t}_a)$ and $\boldsymbol{\theta}_a \equiv \boldsymbol{\theta}(\mathbf{t}_a)$, respectively, and $\mu_{i,a} \equiv \mu_i(\mathbf{t}_a)$ and $m_{j,a} \equiv m_j(\mathbf{t}_a)$. Apart from elements $\kappa_{i,a} \equiv \kappa_i(\mathbf{t}_a)$ and $\theta_{j,a} \equiv \theta_j(\mathbf{t}_a)$, the latter depend on the multilateral resistance terms $\Pi_{i,a}^{-\alpha} \equiv \Pi_i(\mathbf{t}_a)^{-\alpha}$ and $P_{j,a}^{-\alpha} \equiv P_j(\mathbf{t}_a)^{-\alpha}$. We set $P_{1,a}^{-\alpha} = 1/\theta_{1,a}$ as the numéraire, and the equation referring to country 1 as an importer in the system of trade resistances has to be skipped.

Proposition 1 describes the linear approximation of the system of resistance terms and the approximation error. Details on the proof are provided in the Appendix.

Proposition 1. *The linear approximation of the system of multilateral resistances (3) at trade costs $\mathbf{t}_a$ with $\Delta\mathbf{t}_a = \mathbf{t} - \mathbf{t}_a$ allows to write log-normalized exports $\widetilde{\mathbf{x}}(\mathbf{t})$ as*

$$\widetilde{\mathbf{x}}(\mathbf{t}) = \widetilde{\mathbf{x}}_L(\mathbf{t}, \mathbf{t}_a) + \widetilde{\mathbf{r}}(\mathbf{t}, \mathbf{t}_a),$$

where

$$\begin{aligned} \widetilde{\mathbf{x}}_L(\mathbf{t}, \mathbf{t}_a) &= \widetilde{\mathbf{x}}_a + \alpha \mathbf{Q}_a \Delta\mathbf{t}_a \\ \widetilde{\mathbf{r}}(\mathbf{t}, \mathbf{t}_a) &= -\mathbf{D}\left(\mathbf{D}'\mathbf{G}_a\mathbf{D}\right)^{-1} \mathbf{r}(\mathbf{t}, \mathbf{t}_a), \end{aligned}$$

and $\mathbf{Q}_a = \mathbf{I} - \mathbf{P}_a$, $\mathbf{P}_a = \mathbf{D}\left(\mathbf{D}'\mathbf{G}_a\mathbf{D}\right)^{-1}\mathbf{D}'\mathbf{G}_a$ and $\mathbf{G}_a = diag(\widetilde{X}_{ij}(\mathbf{t}_a))$. $\mathbf{I}$ denotes the identity matrix of size N^2 and the $N^2 \times (2N-1)$ design matrix $\mathbf{D}$ collects the binary importer- and exporter-country dummy variables. Lastly, the $2N-1$ vector $\mathbf{r}(\mathbf{t}, \mathbf{t}_a)$ denotes the approximation error of the linearization of the system of multilateral resistance terms.

The linear approximation involves an asymmetric, weighted projection matrix $\mathbf{P}_a$ that projects on the exporter and importer country dummy variables collected in $\mathbf{D}$. At any weight $\widetilde{X}_{ij,a}$ inserted in the diagonal matrix $\mathbf{G}_a$, it holds that $\mathbf{P}_a\mathbf{D} = \mathbf{D}$ and $\mathbf{Q}_a\mathbf{D} = \mathbf{0}$. Hence, the linear approximation transforms the right-hand side variables of the gravity equation, here $\Delta\mathbf{t}_a$, by a weighted two-way within transformation. Note that in linear two-way panel models $\mathbf{G}_a$ is an identity matrix and $\mathbf{P}_a = \mathbf{D}(\mathbf{D}'\mathbf{D})^{-1}\mathbf{D}'$ is symmetric and idempotent (see Baltagi, 2021, chapter 3). In contrast, for a general weighting matrix $\mathbf{G}_a$, $\mathbf{P}_a$ is not symmetric and not idempotent.

The approximation error of the linearized gravity equation, $\widetilde{\mathbf{r}}(\mathbf{t}, \mathbf{t}_a)$, is induced by the linearization of the system of multilateral resistances. In the Appendix we show that this error translates into the approximation error of the gravity equation as $-\mathbf{D}\left(\mathbf{D}'\mathbf{G}_a\mathbf{D}\right)^{-1}\mathbf{r}(\mathbf{t}, \mathbf{t}_a)$. Since the leading term in the last expression is the dummy matrix $\mathbf{D}$, the linearization error of the gravity equation, $\widetilde{\mathbf{r}}(\mathbf{t}, \mathbf{t}_a)$, exhibits unilateral variation only and $\mathbf{Q}_a\widetilde{\mathbf{r}}(\mathbf{t}, \mathbf{t}_a) = \mathbf{0}$.

2.3 Baier and Bergstrand's BvOLS Linearization

Baier and Bergstrand (2009a) linearly approximate the system of multilateral resistances at the point where trade costs are identical across all country pairs (including $i = j$). Using the subscript $a = BB$ for this case, $\mathbf{t}_{ij,BB} = t$ for all i and j.[5] Unlike Baier and Bergstrand (2009a), let us use $\kappa_{i,BB} \equiv \kappa_i(\mathbf{t}_{BB})$ and $\theta_{j,BB} \equiv \theta_j(\mathbf{t}_{BB})$ to indicate the sales and expenditure shares which pertain to that approximation point in equilibrium.[6]

In their equations (15)-(21), Baier and Bergstrand (2009a) derive the approximation of equation (1) under the equilibrium restrictions of multilateral resistances (3) as

$$\ln e^{\alpha t_{ij}}\Pi_i^{-\alpha}P_j^{-\alpha} \approx \alpha\left(t_{ij} - t\right) \tag{4}$$
$$+\alpha\left[-\left(\sum_{k=1}^{N}\theta_k\left(t_{ik} - t\right)\right) - \left(\sum_{k=1}^{N}\kappa_k\left(t_{kj} - t\right)\right) + \left(\sum_{k=1}^{N}\sum_{l=1}^{N}\kappa_k\theta_l\left(t_{kl} - t\right)\right)\right],$$

which does not conform to Proposition 1. The reason is that *observed data* on $\kappa_i \equiv \kappa_i(\mathbf{t})$ and $\theta_j \equiv \theta_j(\mathbf{t})$ are used instead of ones related to the *approximation point*. Note that the system of multilateral resistances (3) is fulfilled at trade costs $\mathbf{t}_{BB}$ only when being combined with the (unobserved) associated equilibrium sales and expenditure shares $\kappa_{i,BB}$ and $\theta_{j,BB}$.

In case of a linear approximation in one dimension, here t_{ij}, all other variables must be measured at the approximation point of the system in order for the linear approximation to be tangential to (i.e., have the same solution as) the nonlinear model at the approximation point. With BvOLS, this is the point where $t_{ij,BB} = t$ for all i and j. Due to a lack of knowledge of $\kappa_i(\mathbf{t}_{BB})$ and $\theta_j(\mathbf{t}_{BB})$, Baier and Bergstrand (2009a) and all applications thereof used observed values $\kappa_i(\mathbf{t})$ and $\theta_j(\mathbf{t})$ instead of the unobserved $\kappa_i(\mathbf{t}_{BB})$ and $\theta_j(\mathbf{t}_{BB})$. This entails a source of bias beyond the approximation bias following from the linearization as such. The mentioned

[5] At symmetric trade costs $\mathbf{t}_{ij,BB} = t$ for all i and j including $j = i$, trade is the same at positive trade costs with $t > 1$ as under free trade with $t = 1$.

[6] Note that the sales and expenditure shares will not be identical across countries due to endowment differences; see equation (2). Hence, differences in sales, expenditures, and exports across countries will prevail also in the absence of trade frictions.

bias depends on the actual variance in trade costs and other fundamentals across countries and country pairs.[7]

Since Baier and Bergstrand (2009a) use sales and expenditure shares measured at $\mathbf{t} \neq \mathbf{t}_{BB}$, they introduce an additional bias beyond the remainder standard linearization error as the following corollary shows.

Corollary 1. *The BvOLS-approximation*
Using the observed sales and expenditure shares $\kappa_i(\mathbf{t})$ and $\theta_j(\mathbf{t})$, the approximation by Baier and Bergstrand (2009a) with subscript BvOLS can be written as

$$\begin{aligned}\widetilde{\mathbf{x}}_{L,BvOLS}(\mathbf{t},\mathbf{t}_{BB}) &= \widetilde{\mathbf{x}}_{BvOLS} + \alpha \mathbf{Q}_{BvOLS}\Delta\mathbf{t}_{BB} \\ \widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t},\mathbf{t}_{BB}) &= (\widetilde{\mathbf{x}}_{BB} - \widetilde{\mathbf{x}}_{BvOLS}) + \alpha\left(\mathbf{Q}_{BB} - \mathbf{Q}_{BvOLS}\right)\Delta\mathbf{t}_{BB} + \widetilde{\mathbf{r}}(\mathbf{t},\mathbf{t}_{BB})\end{aligned}$$

where $\widetilde{\mathbf{x}}_{BvOLS} = diag(\ln(\kappa_i(\mathbf{t})\theta_j(\mathbf{t})))$. $\mathbf{G}_{BvOLS} = diag(\kappa_i(\mathbf{t})\theta_j(\mathbf{t}))$ is used to form the projection matrix $\mathbf{Q}_{BvOLS}$. Lastly, $\widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t},\mathbf{t}_{BB}) = \widetilde{\mathbf{x}}(\mathbf{t},\mathbf{t}_{BB}) - \widetilde{\mathbf{x}}_{L,BvOLS}(\mathbf{t},\mathbf{t}_{BB})$. Thereby terms with subscript BB refer to the theory consistent linear approximation at $\mathbf{t}_{BB} = t\boldsymbol{\iota}_{N^2}$ as derived in Proposition 1. $\boldsymbol{\iota}_{N^2}$ denotes an $N^2 \times 1$ vector of ones.

As shown in the Appendix the linearization bias of BvOLS can be derived using Proposition 1 to split up observed exports into the summed linearized component, $\widetilde{\mathbf{x}}_L(\mathbf{t},\mathbf{t}_{BB})$, and the approximation error $\widetilde{\mathbf{r}}(\mathbf{t},\mathbf{t}_{BB})$. Subtracting $\widetilde{\mathbf{x}}_{L,BvOLS}(\mathbf{t},\mathbf{t}_{BB})$ yields the BvOLS-approximation as described in the Corollary.

Log normalized exports at the point of approximation differ between BvOLS and the suitably linearized model at $t_{ij,BB} = t$ ($\widetilde{\mathbf{x}}_{BvOLS}$ vs. $\widetilde{\mathbf{x}}_{BB}$), as do the projection matrices ($\mathbf{Q}_{BvOLS}$ vs. $\mathbf{Q}_{BB}$). Consequently, neither the approximated trade flows nor the Jacobian of the system of multilateral resistances coincide with their true counterparts of the nonlinear system of multilateral resistances at the point of approximation under BvOLS. Thus, BvOLS does not provide a proper linearization at this point (see Judd, 1998, p. 449). As a result, comparative statics based on the BvOLS model will be biased as well.

[7] See the introduction for a selection of BvOLS-applications. The simulations in Bergstrand, Egger, and Larch (2013) already indicate that there is a nontrivial bias about the BvOLS-approximation when being used for comparative static analysis. In their Table 1, these authors document that there is a trade-cost parameter bias in their BvOLS-2 model, which is exactly the model of Baier and Bergstrand (2009a). In the same table, they report that this bias is absent for what they call BvOLS-1, which is the model of Baier and Bergstrand (2010), avoiding sales- and expenditure-share weights. This is not surprising, as that model is the same as a two-way within estimator (but only for the right-hand side of the model). What we document here, and as is well known, is that BvOLS-1 in Bergstrand, Egger, and Larch (2013) leads to biased standard errors as, unlike the within estimator, the dependent variable is not transformed. Moreover, we show in the present paper that this so-called BvOLS-1 estimator is only suited for parameter-point estimation but not for counterfactual analysis, as it uses identical weights for all countries, irrespective of how large they are.

2.4 A Two-country Example

For the sake of transparency and illustration, let us assume a world of two economies $\{i, j\} = \{1, 2\}$ which have zero domestic trade costs, $t_{ii} = 0$, and symmetric international trade costs, $t_{ij} = t$ for $i \neq j$. With one-sector endowment economies, GDP is defined as $Y_i = p_i A_i$, where p_i is the price per unit of endowment A_i, as charged by the selling country i.[8] We choose country 1's mill price as the numéraire ($p_1 = 1$). To give an example, normalized exports from country 1 to country 2 the BvOLS-approximation at $t_{BB} = 0$ for all i, j yields

$$\begin{aligned}
\widetilde{x}_{12,BB} - \ln \kappa_{1,BB}\theta_{2,BB} &= \alpha t_{12} - \alpha \sum_{k=1}^{2} \theta_{k,BB} t_{1k} - \alpha \sum_{k=1}^{2} \kappa_{k,BB} t_{k2} \\
&+ \alpha \sum_{i=1}^{2} \sum_{k=1}^{2} \kappa_{i,BB}\theta_{k,BB} t_{ik} \\
&= \alpha \left(1 - \theta_{2,BB} - \kappa_{1,BB} + \kappa_{1,BB}\theta_{2,BB} + \kappa_{2,BB}\theta_{1,BB}\right) t. \\
\widetilde{x}_{12,BvOLS} - \ln \kappa_1 \theta_2 &= \alpha t_{12} - \alpha \sum_{k=1}^{2} \theta_k t_{1k} - \alpha \sum_{k=1}^{2} \kappa_k t_{k2} + \alpha \sum_{i=1}^{2} \sum_{k=1}^{2} \kappa_i \theta_k t_{ik} \\
&= \alpha \left(1 - \theta_2 - \kappa_1 + \kappa_1\theta_2 + \kappa_2\theta_1\right) t.
\end{aligned}$$

Under symmetric country sizes and balanced trade, $\kappa_j = \kappa_j = 0.5$ and $\kappa_i = \theta_i$, this reduces to

$$\begin{aligned}
\widetilde{x}_{12,BB} - \ln \kappa_{1,BB}\theta_{2,BB} &= \alpha t, \\
\widetilde{x}_{12,BvOLS} - \ln \kappa_1 \theta_2 &= \alpha t.
\end{aligned}$$

Hence, at perfect symmetry, the BB- and BvOLS-linearizations are identical. Note that this is not the case under asymmetry regarding the sales and expenditure shares at free trade.

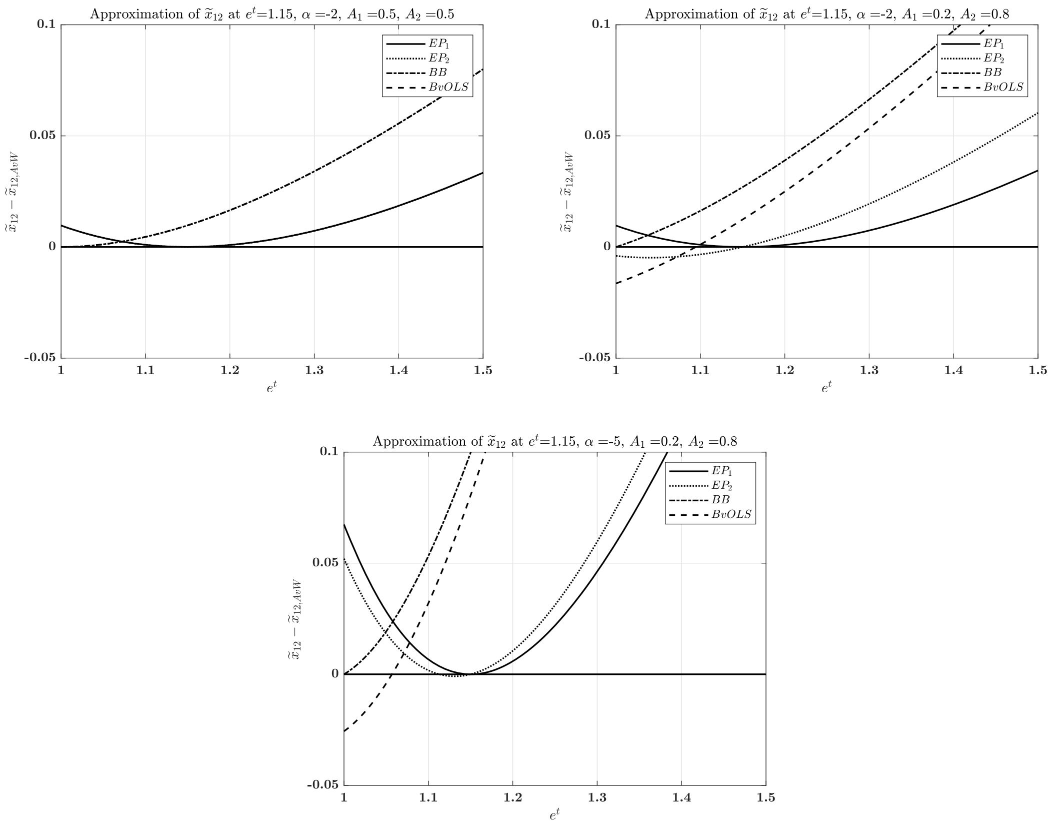
The nonlinear gravity model implies

$$\begin{aligned}
\widetilde{x}_{12,AvW} &= \alpha t_{12} + \ln \mu_1(\mathbf{t}) + \ln m_2(\mathbf{t}) \\
\widetilde{x}_{12,AvW} - \ln \kappa_{1,a}\theta_{2,a} &= \alpha t_{12} + \ln \left(\Pi_1(\mathbf{t})^{-\alpha} P_2(\mathbf{t})^{-\alpha}\right)
\end{aligned}$$

and it requires numerically solving the system of multilateral resistances to obtain $\mu_1(\mathbf{t})$ and $m_2(\mathbf{t})$.

[8] Recall that we consider the case, where the preference parameter called β_i in equation (7) in Anderson and van Wincoop (2003) is unity for both countries. Accordingly, we obtain the relationship that $\alpha \ln p_i(\mathbf{t}) = \ln \mu_i(\mathbf{t})$.

Figure 1: The BvOLS-approximation in a two-country example I



Notes:

BB ... approximation using trade costs t_{BB} and (empirically unobserved) sales and expenditure shares $\{\kappa_{i,BB}, \theta_{j,BB}\}$ pertaining to the approximation point. $BvOLS$... approximation using trade costs t_{BB} and (inappropriately) the observed sales and expenditure shares $\{\kappa_i, \theta_j\}$ pertaining to $t \neq t_{BB}$. EP_1 ... linearization at observed trade costs $t \neq t_{BB}$ and at observed $\{\kappa_i, \theta_j\}$ but (inappropriately) assuming that these shares do not endogenously adjust to changes in t . EP_2 ... as EP_1 but allowing $\{\kappa_i, \theta_j, \}$ to adjust with changing international trade costs, t.

Figures 1-2 illustrate the analytical results for this two-country world under the aforementioned assumptions about trade costs. We present two figures with three panels each to illustrate that the bias we address fundamentally depends on three things (apart from endowments): (i) the level of t; (ii) the gap between the sales and expenditure shares accruing to the approximation point BB and associated with t_{BB}, $\{\kappa_{i,BB}, \theta_{j,BB}\}$, for all $\{i, j\} = \{1, 2\}$ versus the observed ones used by $BvOLS$ and associated with $t = t_{BvOLS} \neq t_{BB}$, $\{\kappa_i, \theta_j\}$;[9] and (iii) the level of the

[9]The difference in international versus domestic trade frictions.

trade elasticity, α.

We consider two configurations each of $\exp(t) \in \{1.15, 1.35\}$ and $\alpha \in \{-5, -2\}$. In each figure, we present four schedules: BB for the approximation using the (empirically unobserved) sales and expenditure shares pertaining to the approximation point $\{\kappa_{i,BB}, \theta_{j,BB}\}$; $BvOLS$ for the BvOLS-approximation which uses the observed $\{\kappa_i, \theta_j\}$ instead of – for a bias-free approximation – the required $\{\kappa_{i,BB}, \theta_{j,BB}\}$; EP_1 for the linearization at observed trade costs and compatible sales and expenditure shares but (inappropriately) assuming that these shares $\{\theta_i, \kappa_i\}$ do not endogenously adjust to changes in t; and EP_2 which on top of EP_1 allows $\{\theta_i, \kappa_i\}$ to adjust to changing international trade costs, t.[10] In all figures, we plot differences between the respective model prediction of log normalized exports of country 1 to country 2, $\widetilde{x}_{12,model}$, where $model \in \{BB, BvOLS, EP_1, EP_2\}$, as a difference to the solution of the true, nonlinear Anderson and van Wincoop (2003) model, $\widetilde{x}_{12,AvW}$.

Let us first discuss the results in Figure 1. Clearly, we see that the difference $\widetilde{x}_{12,model} - \widetilde{x}_{12,AvW}$ depends in an important way on the model type. By design, the intercept is zero for the true Anderson and van Wincoop (2003) model. The models EP_1 and EP_2 touch or intersect, respectively, at their approximation point. The BB-model touches (is identical to) the AvW model at the point $\exp(t_{ij,BB}) = 1$ for $\{i, j\} = \{1, 2\}$, another point than the EP models. Any level of $\widetilde{x}_{12,BB} - \widetilde{x}_{12,AvW}$ to the right of $\exp(t_{ij,BB})$ reflects the true approximation bias of a hypothetical model which relates to the linearization of a nonlinear relationship at otherwise true parameters at $t_{ij,BB} = 0$ for $\{i, j\} = \{1, 2\}$.

Now inspect $model = BvOLS$ as used in practice due to the lack of observations of $\kappa_{i,BB}$ and $\theta_{j,BB}$. First of all, this model does not touch the true nonlinear (AvW) model at the point $t_{ij,BB} = 1$ for $\{i, j\} = \{1, 2\}$. Hence, BvOLS is not tangential to the nonlinear function at the alleged approximation point. In fact, BvOLS approximates the nonlinear function best (intersects with it) at some unknown point t_b, where $t_{BB} < t_b < t$. In any case, BvOLS suffers from a bias beyond the linearization-related approximation bias which purely comes from using $\{\kappa_i, \theta_j\}$ instead of $\{\kappa_{i,BB}, \theta_{j,BB}\}$ in the linearization. The vertical difference $\widetilde{x}_{12,BB} - \widetilde{x}_{12,AvW} | \exp(t) = 1.15$ purely reflects the true linearization bias, whereas $\widetilde{x}_{12,BvOLS} - \widetilde{x}_{12,AvW} | \exp(t) = 1.15$ reflects a conglomerate of the linearization bias and the bias from using $\{\kappa_i, \theta_j\}$ instead of $\{\kappa_{i,BB}, \theta_{j,BB}\}$. Interestingly, the use of the inappropriate sales and expenditure shares counteracts the pure linearization bias of the BB model to some extent.

Both types of linearization proposed here, EP_1 and EP_2, pertain to the point where international trade costs are as realized at $\exp(t) = 1.15$ and sales and expenditure shares are observed at $\{\kappa_i, \theta_j\}$ at this point. The outcome is $\widetilde{x}_{12,model} -$

[10] See Appendix A.5 for details on this approximation.

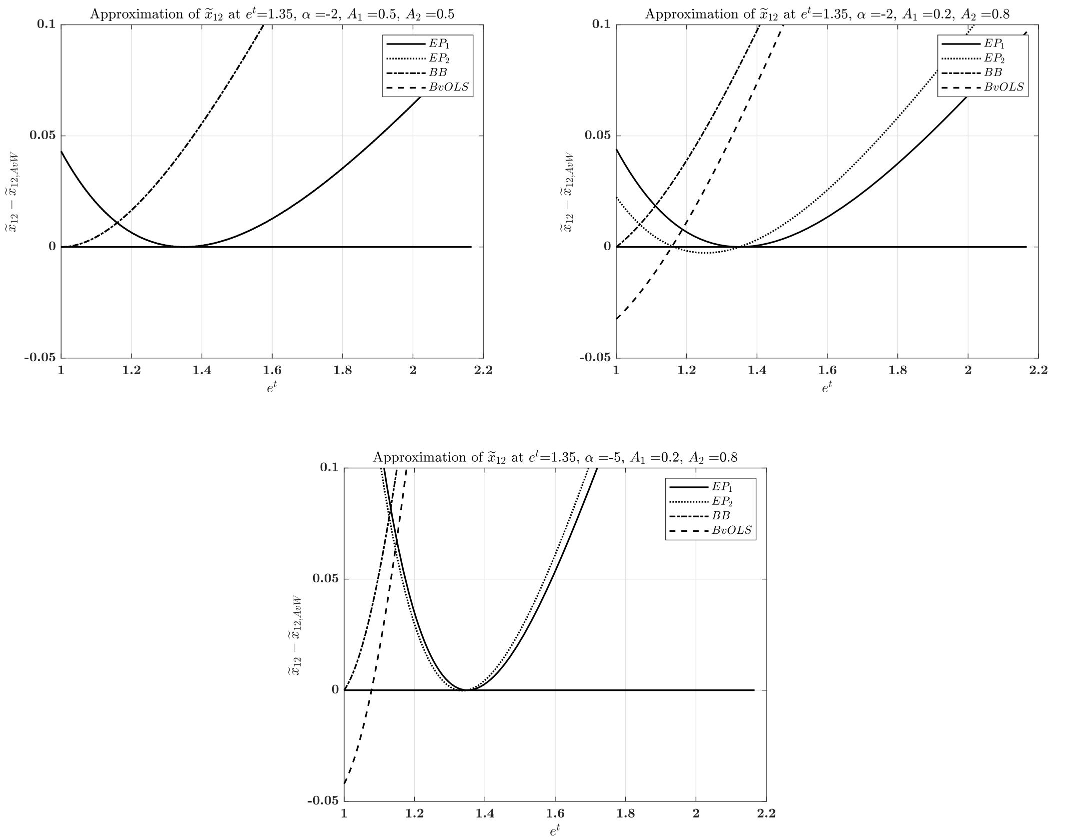
$\widetilde{x}_{12,AvW} = 0$ in this point for either one of these linearizations. There is a slight difference in the slope of the two schedules: $\widetilde{x}_{12,EP_1} - \widetilde{x}_{12,AvW}$ has a minimum in the realization point $\exp(t) = 1.15$, while $x_{12,EP_2} - x_{12,AvW}$ is rotated counterclockwise and does not have a minimum at realization points of $t > 1$. For any $model \in \{BB, BvOLS\}$, $\widetilde{x}_{12,model} - \widetilde{x}_{12,AvW}$ increases at the realization point $\{t_{ij}, \kappa_i, \theta_j\}$ as we increase trade costs from $\exp(t) = 1.15$ in Figure 1. Model EP_1 then generally fares better than all other approximations (including EP_2). However, when reducing trade costs from $\exp(t) = 1.15$ in Figure 1, there is a region in the neighborhood of e^t, where EP_2 outperforms EP_1. At very low trade costs, model BB, and at medium-low trade costs BvOLS, may eventually fare better than EP_1. However, the latter is only due to the nonlinear behavior of linearization biases as we move far away from the approximation point – which involves an additional bias of the BvOLS model.

If realized international trade costs are $\exp(t) = 1.15$ as in Figure 1 but $\alpha = -5$ instead of $\alpha = -2$, the linearization biases become generally larger at sufficiently high values of t. Also the bias involved with BvOLS becomes larger at the (unobserved) approximation point where $\exp(t_{BB}) = 1$ for $\{i, j\} = \{1, 2\}$.

In Figure 2, we conduct the same exercises as in Figure 1, but we set realized international trade costs for $i \neq j$ at $\exp(t) = 1.35$ instead of at $\exp(t) = 1.15$. A comparison of Figure 2 with Figure 1 suggests that the linearization bias of BB and the combined bias of BvOLS become larger in a relatively larger neighborhood around the realized log trade costs as e^t increases.

On a general note, the intercept of the two EP models at $e^t = 1$ is lower than that of either BB or BvOLS at $e^t = 1.15$ in Figure 1 or $e^t = 1.35$ in Figure 2. Hence, the approximation of the implied change of trade costs $\Delta t = t_{ij} - 0$ is closer to the true model under EP than the BB or BvOLS models. The model that fares the worst among all in the two figures in terms of the gap in the function values evaluated at $e^t = 0$ versus $e^t = 1.15$ or $e^t = 1.35$ is BvOLS. However, BvOLS's intercept at realized trade costs is lower than that of BB due to the negative bias at $e^t = 1$.

Figure 2: The BvOLS-approximation in a two-country example II



Notes:

BB ... approximation using trade costs t_{BB} and (empirically unobserved) sales and expenditure shares $\{\kappa_{i,BB}, \theta_{j,BB}\}$ pertaining to the approximation point. $BvOLS$... approximation using trade costs t_{BB} and (inappropriately) the observed sales and expenditure shares $\{\kappa_i, \theta_j\}$ pertaining to $t \neq t_{BB}$. EP_1 ... linearization at observed trade costs $t \neq t_{BB}$ and at observed $\{\kappa_i, \theta_j\}$ but (inappropriately) assuming that these shares do not endogenously adjust to changes in t . EP_2 ... as EP_1 but allowing $\{\kappa_i, \theta_j, \}$ to adjust with changing international trade costs, t.

3 BvOLS- and EP$_1$-model Parameter Estimates and Model Predictions

The following table provides an overview on the various models and the acronyms considered in this paper.

Table 1: Synopsis of considered models

Model (o)	Source	Lineari- zation	Approximation	Approximation point	Note
AvW	AvW (2003)	no	none	none	a)
BB	This paper	yes	MR	free trade	b)
BvOLS	BB (2009a)	yes	MR	mix free trade/obs.	c)
w	2-way within	no	none	none	d)
EP1	This paper	yes	MR	obs.	e)
EP2	This paper	yes	MR and income	obs.	f)

Notes: obs.... observed, MR... multilateral resistance terms.
a) The AvW model estimates trade-cost parameters conditional on multilateral resistance (MR) terms, updates the latter based on the parameters, re-estimates parameters conditional on updated MR terms, etc., until convergence. The approach may be, hence, referred to as one of structurally-iterative estimation.
b) The BB model is the same as BvOLS but it uses the unobservable sales and expenditure shares $\{\kappa_{i,BB}, \theta_{j,BB}\}$ pertaining to the free-trade equilibrium for weighting the log trade costs $t_{ij,BB}$. This is why it is infeasible to do (without simulating the equilibrium model).
c) BvOLS is the incorrectly linearized model that uses free trade with log trade costs $t_{ij,BB}$ as the approximation point but observed sales and expenditure shares $\{\kappa_i, \theta_j\}$ for weighting in the linearization. The latter shares are incompatible with the free-trade equilibrium.
d) The model w is the within estimator which corresponds to a two-way (exporter- and importer-)fixed effects estimator in cross section. This obtains identical parameters to the Baier and Bergstrand (2010) version of BvOLS, which uses $1/N$ instead of $\{\kappa_i, \theta_j\}$ as weights in the linearized MR terms. This model obtains consistent trade-cost parameters but inconsistent standard errors. It keeps the multilateral resistance terms fixed (as fixed effects) in counterfactual equilibrium.
e) EP1 is the linearized model at the observed trade equilibrium, where the observable sales and expenditure shares $\{\kappa_i, \theta_j\}$ are consistent with the log trade costs t_{ij} and the equilibrium trade flows. It delivers consistent trade-cost parameters and can be used for counterfactual analysis. Only the multilateral resistance terms are linearized, whereas $\{\kappa_i, \theta_j\}$ are held fixed.
f) EP2 is a variant of EP1, where also $\{\kappa_i, \theta_j\}$ responses to changes in trade costs are considered. It is also linearized at the observed trade equilibrium, where observable $\{\kappa_i, \theta_j\}$ are consistent with t_{ij}. It delivers consistent trade-cost parameters and can be used for counterfactual analysis.

The econometric specification of the right-hand side of the linearized gravity model comes in logs and augments the systematic part with additive, independent disturbances ε_{ij}, which might be heteroskedastic. Trade barriers are assumed to be exogenous so that $E[\varepsilon_{ij}|t, \boldsymbol{\kappa}, \boldsymbol{\theta}] = \mathbf{0}$.[11] Thus, only exports, but neither trade barriers nor sales or expenditure shares are measured with error, and the system of multilateral resistances holds in expectation as in Anderson and van Wincoop (2003). The true data-generating process can then compactly be described by the empirical model

$$\widetilde{\mathbf{x}}(\mathbf{t}) = \alpha \mathbf{t} + \mathbf{D} \begin{bmatrix} \mathbf{m}(\mathbf{t}) \\ \boldsymbol{\mu}(\mathbf{t}) \end{bmatrix} + \boldsymbol{\varepsilon}.$$

where $\boldsymbol{\mu}(\mathbf{t})$ and $\boldsymbol{m}(\mathbf{t})$ solve the system of multilateral resistances (3). The BvOLS-vector of disturbances is denoted by $\boldsymbol{\varepsilon}_{BvOLS}$ and it includes the remainder approximation error.

The approximated, econometric BvOLS-model in logs reads[12]

$$\Delta\widetilde{\mathbf{x}}_{BvOLS} = \widetilde{\mathbf{x}}(\mathbf{t}) - \widetilde{\mathbf{x}}_{BvOLS} = \alpha \mathbf{Q}_{BvOLS} \Delta \mathbf{t}_{BB} + \boldsymbol{\varepsilon}_{BvOLS},$$

where $\widetilde{\mathbf{x}}_{BvOLS}$ has typical element $\ln \kappa_i(\mathbf{t}) + \ln \theta_j(\mathbf{t})$ and $\Delta \mathbf{t}_{BB}$ has typical element $t_{ij} - t_{BB}$. The following proposition is proven in the Appendix.

Proposition 2. *(i) The BvOLS estimate $\widehat{\alpha}_{BvOLS}$ and its deviation from the true parameter α are given by*

$$\begin{aligned} \widehat{\alpha}_{BvOLS} &= \mathbf{H}^{-1}_{BvOLS} \Delta \mathbf{t}'_{BB} \mathbf{Q}'_{BvOLS} \Delta \widetilde{\mathbf{x}}_{BvOLS}. \\ \widehat{\alpha}_{BvOLS} - \alpha &= \mathbf{H}^{-1}_{BvOLS} \Delta \mathbf{t}'_{BB} \mathbf{Q}'_{BvOLS} \left[\widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t}, \mathbf{t}_{BB}) + \boldsymbol{\varepsilon} \right], \end{aligned}$$

where we define $\mathbf{H}_{BvOLS} = \Delta \mathbf{t}'_{BB} \mathbf{Q}'_{BvOLS} \mathbf{Q}_{BvOLS} \Delta \mathbf{t}_{BB}$.
(ii) Under exogenous trade barriers, the bias of $\widehat{\alpha}_{BvOLS}$ is given as

$$\begin{aligned} Bias_{BvOLS} &\equiv E[\widehat{\alpha}_{BvOLS} - \alpha | \mathbf{t}, \boldsymbol{\kappa}, \boldsymbol{\theta}] \\ &= \mathbf{H}^{-1}_{BvOLS} \Delta \mathbf{t}'_{BB} \mathbf{Q}'_{BvOLS} \widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t}, \mathbf{t}_{BB}) \neq 0, \end{aligned}$$

(iii) The unweighted BvOLS estimator of α with subscript w uses $\mathbf{G}_w = \mathbf{I}_{N^2}$, an $N^2 \times N^2$ identity matrix to form the projection matrix $\mathbf{Q}_w$, and it is unbiased.
(iv) Assuming that the disturbances are independent but heteroskedastic with diagonal variance-covariance matrix $\boldsymbol{\Omega}_\varepsilon$, it follows that

$$\begin{aligned} &E[(\widehat{\alpha}_{BvOLS} - \alpha - Bias_{BvOLS})^2 | \mathbf{t}, \boldsymbol{\kappa}, \boldsymbol{\theta}] \\ &= \mathbf{H}^{-1}_{BvOLS} \Delta \mathbf{t}'_{BB} \mathbf{Q}'_{BvOLS} \boldsymbol{\Omega}_\varepsilon \mathbf{Q}_{BvOLS} \Delta \mathbf{t}_{BB} \mathbf{H}^{-1}_{BvOLS}. \end{aligned}$$

[11] A modification of the approach to include endogenous trade-cost variables would be straightforward and could follow a customary instrumental-variables approach.

[12] Baier and Bergstrand (2009a) use $\Delta \mathbf{x}$ rather than normalized trade flows $\Delta\widetilde{\mathbf{x}}$ and include a constant to account for total world expenditures. This constant implicitly absorbs the average approximation error in addition to log world GDP.

This proposition shows that the BvOLS-approximation does not lead to an unbiased estimator of the trade-elasticity parameter α. The reason lies in the usage of the asymmetric projection matrix $\mathbf{Q}_{BvOLS}$ with $\mathbf{Q}'_{BvOLS} \neq \mathbf{Q}_{BvOLS}$, so that $\mathbf{Q}'_{BvOLS}\mathbf{Q}_{BvOLS} \neq \mathbf{Q}_{BvOLS}$ and $\mathbf{Q}'_{BvOLS}\mathbf{D} \neq \mathbf{0}$. It follows that $\mathbf{Q}'_{BvOLS}\mathbf{r}_{BvOLS}(\mathbf{t}, \mathbf{t}_{BB}) \neq \mathbf{0}$ as well. Generally, there is no possibility to estimate the size of the bias in order to establish a bias-corrected estimator of α under BvOLS without solving the nonlinear system of multilateral resistance terms.

However, at $\mathbf{G}_w = \mathbf{I}_{N^2}$ this asymmetry disappears, since the projection matrix is now given by $\mathbf{Q}_w = \mathbf{I} - \mathbf{D}(\mathbf{D}'\mathbf{D})^{-1}\mathbf{D}'$, which is symmetric and idempotent. It is straightforward to verify (see the Appendix) that $\mathbf{Q}_w\widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t}, \mathbf{t}_{BB}) = \mathbf{0}$.[13] The latter approach has been proposed by Baier and Bergstrand (2010, p. 103), who consider the approximation of the system of multilateral resistances at a fully symmetric world where all countries exhibit the same size (GDP) and trade costs are identical (including for domestic sales). This eliminates the approximation error at the approximation point. However, this approximation point is highly unrealistic from an economic theory point of view and large approximation errors of predicted trade flows have to be expected, when applying it with real-world data. The reason is that countries differ starkly in their GDPs (compare Liechtenstein or Belgium with China or the United States) as well as in trade costs (compare the average distance of Austria versus that of Australia to other countries). However, the (sales-and-expenditure-share-) unweighted approach is useful at least for estimation, as it avoids the bias in the trade-elasticity parameter, unlike the BvOLS-approach of Baier and Bergstrand (2009a). Lastly, the proposition also suggests that, besides yielding biased estimates, under BvOLS the estimated standard errors of $\widehat{\alpha}_{BvOLS}$ and inference will be incorrect as well.

Consider a counterfactual change in the trade frictions of $\mathbf{t}$ to $\mathbf{t}_c$ and assume that both the base and counterfactual are predicted with BvOLS (i.e., both are based on the linearization at $t_{BB} = t\boldsymbol{\iota}_{N^2}$). With BvOLS, neither approximated trade flows nor the Jacobian coincide with their counterparts of the nonlinear system of trade resistances at the point of approximation, which is the baseline prediction of observed trade flows both for the base and the counterfactual. As a result, comparative statics based on BvOLS will be biased as well.

With the EP_1 model, we predict the bilateral trade flows with a linearization at observed trade costs $\mathbf{t}$, using the unbiased within estimator and the estimated fixed effects to estimate the normalized multilateral resistance terms, i.e., the approximation point at $\widetilde{x}_{ij,EP_1} = \hat{\alpha}_w t_{ij} + \hat{\mu}_i(\mathbf{t}) + \hat{m}_j(\mathbf{t})$. The typical element, $\widetilde{X}_{ij,EP_1}$, is readily available from country-fixed-effects estimation and can be estimated by the predicted trade flows under observed trade frictions. In this case, $\mathbf{G}_{EP_1} = diag(\widetilde{X}_{ij,EP_1})$ is used to form the projection matrix $\mathbf{Q}_{EP_1}$. We suspect that

[13] In this setting, the assumed trade and expenditure shares each amount to $1/N$.

the approximation bias of any comparative static change in trade costs in the vicinity of what is observed will be smaller than that obtained under the BvOLS-approximation. Specifically, at $\mathbf{t}_a = \mathbf{t}$ the approximation error disappears and the prediction of the observed baseline trade flows is free of any approximation error.

Ignoring the associated change in sales and expenditure shares in a first step, the following corollary shows the prediction error of the counterfactual change in trade flows induced by the BvOLS-linearization of the system of multilateral resistances.

Corollary 2. *Counterfactual prediction*
Denote the vector of counterfactual trade frictions by $\mathbf{t}_c$ and the observed ones by $\mathbf{t}$.
(i) The expected prediction error of the counterfactual change in log exports under BvOLS amounts to

$$E\left[\left(\widehat{\widetilde{\mathbf{x}}}^c_{BvOLS} - \widehat{\widetilde{\mathbf{x}}}_{BvOLS}\right) - (\widetilde{\mathbf{x}}^c - \widetilde{\mathbf{x}})\,|\mathbf{t}, \mathbf{t}_c, \boldsymbol{\kappa}, \boldsymbol{\theta}\right]$$
$$= \mathbf{Q}_{BvOLS}\,(\mathbf{t}^c - \mathbf{t})\, Bias_{BvOLS} - \widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t}^c, \mathbf{t}_{BB}) + \widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t}, \mathbf{t}_{BB}).$$

(ii) Approximating at observed trade costs $\mathbf{t}$ with the EP_1 model, the expected prediction error is

$$E\left[\left(\widehat{\widetilde{\mathbf{x}}}^c_{EP_1} - \widehat{\widetilde{\mathbf{x}}}_{EP_1}\right) - (\widetilde{\mathbf{x}}^c - \widetilde{\mathbf{x}})\,|\mathbf{t}, \mathbf{t}_c, \boldsymbol{\kappa}, \boldsymbol{\theta}\right] = -\widetilde{\mathbf{r}}(\mathbf{t}^c, \mathbf{t}).$$

The BvOLS-prediction for both the base and counterfactual uses the same approximation point and it is prone to two approximation errors, $\widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t}^c, \mathbf{t}_{BB})$ and $\widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t}, \mathbf{t}_{BB})$. In addition, it suffers from a biased estimate of the trade friction parameter, $\widehat{\alpha}_{BvOLS}$ (or the parameter vector on observable trade-cost measures, $\widehat{\boldsymbol{\zeta}}_{BvOLS} = (\widehat{\alpha\boldsymbol{\delta}})_{BvOLS}$). As a result, neither the trade flows corresponding to the base nor those in the counterfactual obey the equilibrium conditions imposed by the system of trade resistances.

The second part of the corollary indicates that predicting both base and counterfactual based on the unbiased within estimator of α, $\hat{\alpha}_w$ (or $\widehat{\boldsymbol{\zeta}}_w$) for observable trade-cost measures), and linearizing the counterfactual prediction in the base at observed trade costs $\mathbf{t}$ is a preferable alternative (using subscript EP for the aforementioned approximation). This approximation uses a different within-projection matrix from BvOLS, namely one that is based on $\mathbf{G}_{EP_1} = diag(\widehat{\widetilde{\mathbf{X}}}_{EP_1})$. Forming $\mathbf{Q}_{EP_1}$ is straightforward. The corollary shows that the expected prediction error of the EP_1 approximation only includes one term, namely the approximation error of the counterfactual $-\widetilde{\mathbf{r}}(\mathbf{t}^c, \mathbf{t})$. We presume that the expected prediction error is smaller than that under BvOLS.

To sum up, BvOLS suffers from two biases, an approximation bias and a parameter-estimation bias. Monte Carlo simulations can illustrate their relative magnitude in real-world data situations.

4 Monte Carlo Simulations Based On Real-world Data

We use data on aggregated exports of manufactured goods among $N = 42$ countries from WIOD for the year 2012. We consider trade frictions such as distance, contiguity and common language from CEPII (see Mayer and Zignago, 2011) and information on membership in regional trade agreements (RTAs) from Mario Larch's Regional Trade Agreements Database described in Egger and Larch (2008).[14]

In a first step, we use this database to set up the data-generating process (DGP) underlying the Monte Carlo simulations below. Specifically, we use a Poisson pseudo-maximum-likelihood model with fixed exporter and importer effects and a log-additive trade-cost function to generate true trade costs from the export data. This obtains the conditional mean as well as the residuals. We draw from a stochastic process to generate the disturbances in $20,000$ Monte Carlo samples, always using the same conditional mean. We then assess the average bias and root mean-squared error of specifically of BB and BvOLS, and compare them to the true-by-construction, nonlinear AvW-estimator and the within (fixed-country-effects) estimator in a controlled environment that mimics the real world. Thereby we focus on statistics about the parameters of the variables in the trade-cost function.

In a second step, we investigate the predictive performance of BB and BvOLS both for baseline trade flows and for ones associated with counterfactually changed trade costs, highlighting the effects of a reduction in ad-valorem trade costs. For comparison, we use the average of the simulated parameter estimates for each estimator based on the $20,000$ Monte Carlo draws.

In order to obtain a true model for the DGP against which the linearization bias can be measured, we first estimate the following standard gravity model by

[14] Among the trade-cost variables, $Log\ distance_{ij}$, the great-circle distance between countries i and j, is the only continuous measure. All other variables are binary indicators. $Border_{ij}$ is a dummy that is unity, if trade flows cross the country border ($i \neq j$), and zero otherwise ($i = j$). $Contiguity_{ij}$ indicates whether two countries i and $j \neq i$ have a common land border or not. $Common\ language_{ij}$ indicates a common official language between countries i and $j \neq i$. Finally, RTA_{ij} indicates membership in the same preferential trade agreement for countries i and $j \neq i$. For domestic sales ($i = j$), all binary indicators are coded as zero. $Log\ distance_{ij}$ assumes the (non-zero) value provided in the CEPII data for domestic sales.

Poisson pseudo-maximum likelihood (PPML):

$$\widetilde{X}_{ij} = \exp(\mathbf{z}'_{ij}\boldsymbol{\zeta} + \beta_i + \gamma_j + \varepsilon_{ij}), \tag{5}$$

where the explanatory trade-cost variables are collected in the vector $\mathbf{z}_{ij}$, and $\boldsymbol{\zeta}$ denotes the corresponding parameter vector.

By design, $\sum_{i=1}^{N}\sum_{j=1}^{N}\widetilde{X}_{ij} = 1$. PPML-estimation with fixed exporter and importer effects implies that the predicted values are consistent with the system of multilateral resistances specified as

$$\begin{aligned} \kappa_i &= \sum_{j=1}^{N}\exp(\mathbf{z}'_{ij}\widehat{\boldsymbol{\zeta}} + \widehat{\beta}_i + \widehat{\gamma}_j) && (6) \\ \theta_j &= \sum_{j=1}^{N}\exp(\mathbf{z}'_{ij}\widehat{\boldsymbol{\zeta}} + \widehat{\beta}_i + \widehat{\gamma}_j) && (7) \end{aligned}$$

with $e^{\widehat{\beta}_i} = \kappa_i\widehat{\Pi_i^{-\alpha}}$, $e^{\widehat{\gamma}_j} = \theta_j\widehat{P_j^{-\alpha}}$ and

$$\begin{aligned} \widehat{P_j^{\alpha}} &= \sum_{i=1}^{N}\exp(\mathbf{z}'_{ij}\widehat{\boldsymbol{\zeta}})\kappa_i\widehat{\Pi_i^{-\alpha}}, \quad j = 2, ..., N \\ \widehat{\Pi_i^{\alpha}} &= \sum_{j=1}^{N}\exp(\mathbf{z}'_{ij}\widehat{\boldsymbol{\zeta}})\theta_j\widehat{P_j^{-\alpha}}, \quad i = 1, ..., N. \end{aligned}$$

Taking the estimates of $\boldsymbol{\zeta}$ as the true values and augmenting the gravity model with a stochastic disturbance term ε_{ij} gives the gravity specification in logs that will be used as the DGP in the subsequent $20,000$ Monte Carlo simulations :

$$\widetilde{x}_{ij} = \mathbf{z}'_{ij}\widehat{\boldsymbol{\zeta}} + \widehat{\beta}_i + \widehat{\gamma}_j + \varepsilon_{ij} = \widetilde{x}^*_{ij} + \varepsilon_{ij}. \tag{8}$$

We follow Borusyak, Jaravel, and Spiess (2021) to use the wild bootstrap to design the DGP of the disturbances. The main advantage of the wild bootstrap lies in its ability to mimic the distribution of the unobserved disturbances in empirical applications in a very realistic way.[15] First, we postulate

$$\varepsilon_{ij} = e_{ij}\xi_{ij}.$$

[15] In an alternative setting we considered, the disturbances ε_{ij} were a random draw from the normal distribution with mean 0 and variance $\sigma_{ij}^2 = exp(0.25d_{ij})/N^{-2}\sum_{i=1}^{N}\sum_{j=1}^{N}exp(0.25d_{ij}))$, where d_{ij} denotes the log distance between countries i and j, to make the disturbances heteroskedastic. The average of σ_{ij} was normalized to 0.72 to match the average standard error of the wild bootstrap disturbances. The respective Monte Carlo simulation results are very similar to the ones in focus here and reported in the Appendix B.

where e_{ij} denotes the OLS residuals of the gravity model in logs with fixed exporter and importer effects as defined in equation (8). In the next step we specify the wild bootstrap disturbances (see Hansen, 2022). We hold e_{ij} as well as the explanatory variables, $\mathbf{z}_{ij}$, and the corresponding multilateral resistance terms fixed in repeated samples across the 20,000 Monte Carlo runs. In each run we draw ξ_{ij} from the Rademacher distribution with ξ_{ij}, taking the values 1 and -1 with probability 1/2, respectively. This implies that $E[\xi_{ij}] = 0$ and $Var[\xi_{ij}] = 1$. The conditional variance of ε_{ij} is thus $Var[\varepsilon_{ij}|e_{ij}] = e_{ij}^2$ and reflects the pattern of heteroskedasticity in the stochastic gravity model.

Table 1 reports on the approximation errors of the BB- and BvOLS-approximations of the systematic part of the structural gravity model at true parameters as used to set up the DGP for the Monte Carlo simulations ignoring the disturbances, $\widetilde{x}_{ij}^*$. In a first column, Table 1 reports on the quintiles of the true trade flows consistent with the Anderson and van Wincoop (2003) model (AvW). The table displays the corresponding approximation errors in percent as well as the decomposition of the BvOLS-approximation error as derived in Corollary 1.

Two results stand out. First, both the BB- and BvOLS-approximations exhibit a negative and substantial approximation error amounting to -63.2 (BB) and -66.7 (BvOLS) percent of the true export flows on average. Moreover, the approximation error increases with the size of the trade flows in both cases with the BvOLS-linearization error turning out somewhat larger than that of BB in all quintiles. For the BB-approximation, the difference between the fifth and the first quintile amounts to -22.7 percentage points. This difference is similar for the BvOLS-approximation (-22.6 percentage points).

Second, the decomposition of the bias as in Corollary 1 reveals that the difference in the sales and expenditure shares (true ones under BB versus observed ones under BvOLS) contributes 12 percent on average to the BvOLS-approximation error. This contribution decreases with the size of the true exports across quintiles, varying between 16.7 percent in the lowest quintile and 6.8 in the highest one. In contrast, the usage of different projection matrices in the penultimate column plays a minor role.

Table 2: The bias of the BvOLS-approximation and its decomposition

Quintiles	$\widetilde{x}_{ij,AvW}$	$\widetilde{\mathbf{r}}(\mathbf{t},\mathbf{t}_{BB})$	$\widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t},\mathbf{t}_{BB})$	In percent of $\widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t},\mathbf{t}_{BB})$		
				$\widetilde{\mathbf{x}}_{BB}-\widetilde{\mathbf{x}}_{BvOLS}$	$\alpha\left(\mathbf{Q}_{BB}-\mathbf{Q}_{BvOLS}\right)\Delta\mathbf{t}_{BB}$	$\widetilde{\mathbf{r}}(\mathbf{t},\mathbf{t}_{BB})$
1	−13.16	−47.86	−53.14	16.70	−3.00	86.30
3	−11.38	−59.50	−63.35	14.20	−3.50	89.30
3	−10.39	−65.60	−68.98	12.30	−2.70	90.30
4	−9.43	−69.55	−72.54	10.00	−1.80	91.90
5	−7.72	−73.52	−75.69	6.80	−0.80	94.10
Total	−10.42	−63.20	−66.73	12.00	−2.40	90.40

Notes: Index AvW pertains to the true Anderson and van Wincoop (2003) model. Indices BB and $BvOLS$ refer to models that assume that trade costs in the approximation point are $\mathbf{t}_{BB} = t\boldsymbol{\iota}_{N^2}$ and sales and expenditure shares are consistent with that or as observed, respectively. The columns labelled $\widetilde{\mathbf{r}}(\mathbf{t},\mathbf{t}_{BB})$ and $\widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t},\mathbf{t}_{BB})$ contain the BB- and BvOLS-approximation errors in percent of the true AvW-values. The last three columns are the components in percent of $Bias_{BvOLS}$ as derived in Corollary 1. There are 42 countries and 1,764 country-pair observations. Quintiles refer to the predicted AvW-trade flows.

Equipped with this artificial theory-consistent data set, the Monte Carlo exercise proceeds with three types of estimators:

1. The nonlinear Anderson and van Wincoop (2003) estimator (AvW), which imposes the system of multilateral resistances as a restriction assuming the equilibrium conditions hold in expectation. The system is solved in an inner loop to obtain estimates $\widehat{\beta}_i$ and $\widehat{\gamma}_j$ at given $\widehat{\boldsymbol{\zeta}}$. The outer loop estimates $\widehat{\boldsymbol{\zeta}}$ by OLS for given $\widehat{\beta}_i$ and $\widehat{\gamma}_j$. Similar to the PPML estimator with country-fixed effects, this estimator obtains theory-consistent parameter estimates that adhere to the system of multilateral resistances under the adopted assumptions.

2. Using either the observed sales and expenditure shares (BvOLS) or the theory-consistent ones (BB) to form the weighted within transformation matrices $\mathbf{Q}_{BvOLS}$ or $\mathbf{Q}_{BB}$, respectively, BvOLS- or BB-estimation proceed as described in Sections 2 and 3. BvOLS and BB include a constant to obtain properly centered residuals. Hence, the specifications control for the average approximation bias.

3. The within estimator (model w) uses the unweighted projection matrix $\mathbf{Q}_w = \mathbf{I} - \mathbf{D}(\mathbf{D}'\mathbf{D})^{-1}\mathbf{D}'$ to sweep the exporter- and importer-country fixed effects and then applies OLS.

Table 2 reports the simulated deviations of the estimated parameters under $model \in \{AvW, BB, BvOLS, w\}$ from their true counterparts measured as the bias in percent. The corresponding results indicate that the choice of the model indeed affects trade-cost parameter estimation.

First, there is virtually no bias of the AvW and the within estimates, as one would expect in general and from the analysis above. In terms of the mean-squared error, there is no advantage of using the nonlinear AvW estimator over the within estimator.

Second, both the BB- and BvOLS-estimates are prone to substantial biases. The largest biases are found for the impact of contiguity with the bias being as large as 39.2 percent with BvOLS and even 53.7 percent with BB in absolute value, respectively. The bias of BvOLS exceeds 9 percent and the one of BB 8 percent for all parameters in Table 2. The BvOLS-parameter estimates are better off than the BB estimates that are based on the theory-consistent approximation point at $t_{ij,a} = t$ for all i and j in almost all cases, except for log distance. Although the average approximation error of BvOLS is slightly larger in absolute value than that of BB as indicated by the results in Table 1, this does not translate one-for-one into a parameter bias as derived in Proposition 2. Having a closer look at the components of the simulated parameter bias points to an important impact of the difference of the approximation points in shaping that bias. It works in the opposite direction of the BB-linearization bias and, hence, reduces the simulated bias of the BvOLS-parameter estimates.

Third, in terms of the root mean-squared error the difference between the BB- and BvOLS-estimators turns out relatively small. Only the estimated border-dummy parameter forms an exception. The corresponding root mean-squared error under BB amounts to 0.51, while for BvOLS we find a smaller value of 0.33. Overall, the findings in Table 2 are well in line with some earlier results, see, e.g., Bergstrand, Egger, and Larch, (2013, Table 1).

Table 3: Monte Carlo simulation results

Bias in percent	ζ_{AvW}	ζ_{BB}	ζ_{BvOLS}	ζ_{w}
Border	-0.04	21.53	11.96	-0.02
Log distance	0.06	8.62	9.33	-0.06
Contiguity	-0.73	-53.72	-39.36	-0.26
Common language	0.37	-39.21	-26.18	0.27
RTA	0.42	20.16	19.46	0.10
Root mean-squared error				
Border	0.24	0.51	0.33	0.26
Log distance	0.05	0.08	0.08	0.05
Contiguity	0.11	0.18	0.14	0.10
Common language	0.13	0.17	0.14	0.12
RTA	0.06	0.13	0.13	0.09

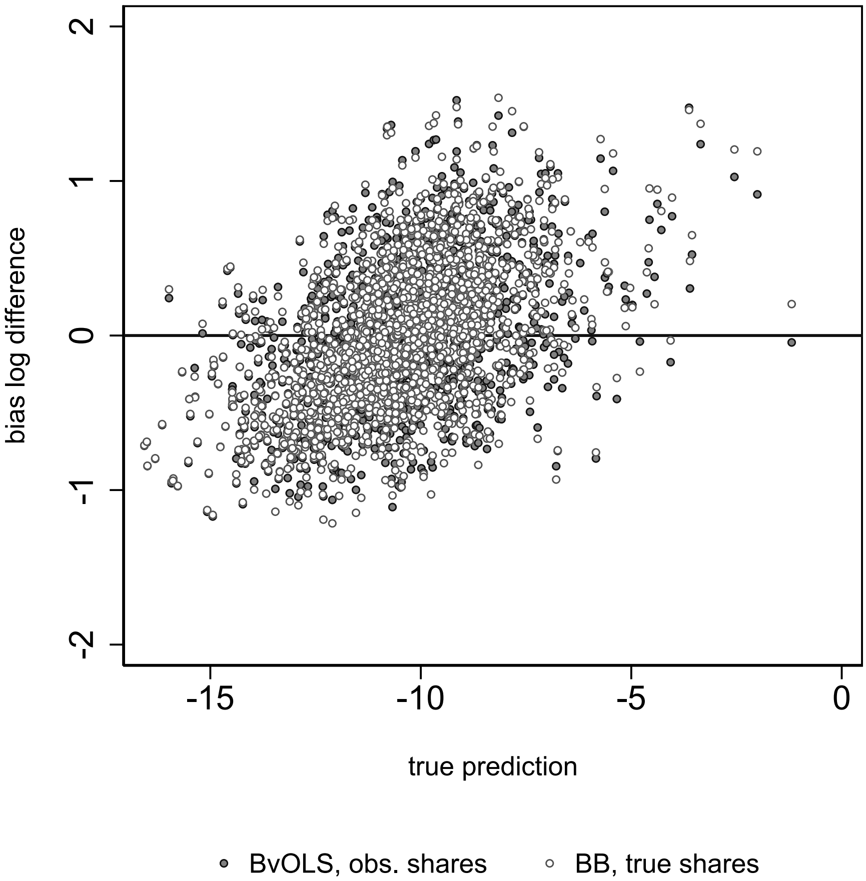
Notes: Index AvW pertains to the true Anderson and van Wincoop (2003) model. $BvOLS$ indicates the usage of observed expenditure and sales shares in forming the weighted within transformation matrix, while BB stands for the use of true shares at the approximation point. Subscript w indicates two-way (unweighted within) country-fixed-effects parameters. There are 42 countries, 1,764 country-pair observations, and 20,000 Monte Carlo runs.

In order to provide further evidence on the approximation errors, we average the simulated parameter estimates over all Monte Carlo runs. Based on these means we calculate the implied predictions of the trade flows at the observed trade frictions and plot the log differences between these predictions of the linear approximations against the truth as described by the nonlinear general-equilibrium model. Since the trade-friction parameters are treated as fixed and all predictions are free of the stochastic disturbances, prediction errors only reflect the biases of the estimated parameters and the approximation errors together.

The AvW predictions for the observed baseline are unbiased up to a small numerical margin. Hence, they are the preferred ones for counterfactual analysis. For the EP_1-predictions (not displayed in the graphs), the same holds true. Note that the EP_1-approximation uses within-model predictions of $\widetilde{x}^*_{ij}$ based on equation (8) to form the baseline predictions of the observed log exports. Since the predicted trade frictions serve as the point of approximation, the EP_1-prediction is free of an approximation bias at that point so that there is no need for illustration.

In contrast, we observe substantial deviations from the truth for the BB- and BvOLS-approximations as shown in Figure 3. The weighting scheme of the projection matrices used to derive the predictions is very similar in both cases, and the different points of approximation do not make a sizeable difference as compared to the overall linearization error. Overall, both BB and BvOLS tend to produce upward-biased predictions for larger bilateral trade flows and downward-biased ones for smaller bilateral trade flows. This finding is line with the findings in Table 1.[16]

Figure 3: The bias of the baseline predictions: BvOLS and BB models vs. true value



Note: Both models include a constant that absorbs the average linearization error.

[16]We acknowledged above that two BvOLS versions had been proposed by Baier and Bergstrand (2009a, 2010), one weighs trade costs by sales and expenditure shares and the second one does not (putting a weight of $1/N$ where N is the number of countries). The "unweighted" approach of Baier and Bergstrand (2010) lacks the parameter bias (akin to a two-way country-fixed-effects estimator) relative to weighted BvOLS in Baier and Bergstrand (2009a). However, for trade-flow predictions or counterfactual analysis, the weighted BvOLS approach outperforms the unweighted one in spite of all its biases. Therefore, we suppress a presentation of the results for the unweighted BvOLS approach in the remainder of the paper.

BvOLS leads to severely biased predictions of counterfactual changes. We illustrate this point by focusing on two a counterfactual reduction in ad-valorem trade cost and report changes in counterfactual log outcome, $\hat{\tilde{x}}^c_{ij}$, relative to (minus) benchmark outcome, $\hat{\tilde{x}}_{ij}$. Specifically, for any $model \in \{BvOLS, EP_1\}$, we compute $\Delta\hat{\tilde{x}}_{ij,model} = \hat{\tilde{x}}^c_{ij,model} - \hat{\tilde{x}}_{ij,model}$ and $\Delta\hat{\tilde{x}}_{ij,model} - \Delta\tilde{x}_{ij,AvW}$, i.e., the difference in predicted comparative-static changes in a linearized model relative to the true (Anderson and van Wincoop) nonlinear model.

We conduct an experiment with two alternative magnitudes of trade-cost changes. Specifically, we choose a counterfactual-minus-benchmark change at observed trade costs of $t^c_{ij} - t_{ij} \in \{0.95; 0.75\}$.[17] The reason for considering two magnitudes of change is that larger changes involve bigger linearization biases (see Section 2.4).

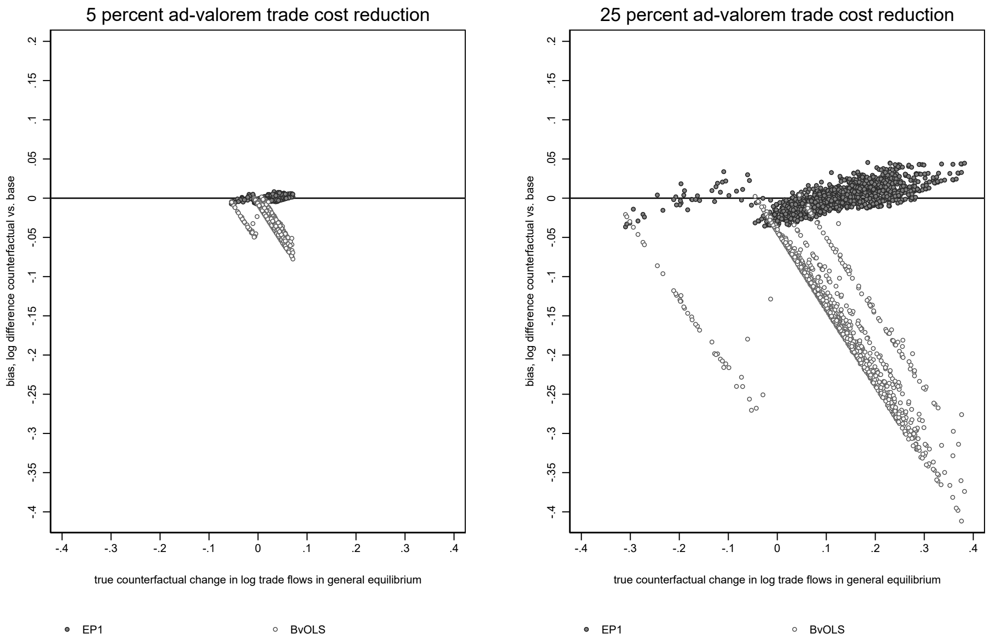
The results for this experiment and the associated magnitudes of change for log normalized trade flows in the BvOLS and EP_1 models in deviation of the true nonlinear model (AvW) are summarized in two panels in Figure 4. The left panel involves a reduction of ad-valorem trade costs by 5% and the one on the right by 25%.

Prior to looking at the figure, let us form expectations in terms of the insights from Section 2.4 above. Note that the considered counterfactual changes correspond to ones where trade costs are reduced from the point where the EP_1 model is tangential to the horizontal axis in Figures 1-2. Reducing trade costs there suggests that EP_1 should predict the change $\Delta\tilde{x}_{ij,AvW}$ quite well and with a small positive bias which increases with a larger considered change in trade costs. Hence, we would expect $\Delta\hat{\tilde{x}}_{ij,EP_1} - \Delta\tilde{x}_{ij,AvW}$ to be mildly positive and larger the larger is $\mathbf{t}^c - \mathbf{t}$. At the considered reference point at $\mathbf{t} \neq t\boldsymbol{\iota}_{N^2}$, $\Delta\hat{\tilde{x}}_{ij,BvOLS} - \Delta\tilde{x}_{ij,AvW}$ is relatively steep in Figures 1 and 2. It is positively sloped to the left of the reference-point t_{ij}, whereas $\Delta\hat{\tilde{x}}_{ij,EP_1} - \Delta\tilde{x}_{ij,AvW}$ is negatively-sloped. Finally, in the vicinity of $\mathbf{t}$, the slope of $\Delta\hat{\tilde{x}}_{ij,BvOLS} - \Delta\tilde{x}_{ij,AvW}$ in absolute value is much bigger than that of $\Delta\hat{\tilde{x}}_{ij,EP_1} - \Delta\tilde{x}_{ij,AvW}$. Combining these insights, we would expect a potentially large, negative value of $\Delta\hat{\tilde{x}}_{ij,BvOLS} - \Delta\tilde{x}_{ij,AvW}$ and a small, positive one for $\Delta\hat{\tilde{x}}_{ij,EP_1} - \Delta\tilde{x}_{ij,AvW}$.

An inspection of Figure 4 corroborates all of the above expectations, while additionally attesting to a substantial variation in the magnitude of the biases, especially for BvOLS, across country pairs.

[17] One could alternatively change specific observables behind trade costs such as distance, but such an experiment would be proportional to the one chosen. However, in such an experiment the predicted counterfactual change would additionally affected by the bias of the estimated trade cost parameters under BvOLS.

Figure 4: The bias of the estimated impact of a counterfactual ad-valorem trade cost change on predicted bilateral flows



Note: The dark points refer to the EP_1-approximation using the unweighted within estimates of the trade friction parameters. The approximation point is at observed trade costs. The light gray points represent the bias of the counterfactual BvOLS predictions.

Table 3 focuses on the latter and reports on average biases (mean) as well as standard deviations (sd) of the comparative-static changes in $\Delta\hat{\tilde{x}}_{ij,model}$ relative to the true nonlinear model in the respective country group. For cross-border trade we consider four groups of country pairs: ones where both the exporter's and the importer's sales are above the median country (large-large), ones where only the exporter or importer is large in terms of of sales (large-small, small-large), and ones where both the exporter and the importer are small in terms of unilateral exports (small-small). We specifically report on two linearized models, namely BvOLS and EP_1. The table also displays results on the counterfactual change in domestic sales for small and large countries. The results in Table 3 contain averages of true predicted counterfactual changes by the AvW model (AvW, mean), and the average biases and their standard deviations across country pairs for BvOLS and EP_1 in two subsequent pairs of columns.

Table 3 shows that EP_1 exhibits a small, negligible bias of predicted counterfactual changes in bilateral exports in all considered groups of countries. As expected, the bias is somewhat larger with a bigger considered change in trade costs (25% versus 5%). The bias is larger by at least one order of magnitude with BvOLS versus EP_1. In particular, the bias of BvOLS is larger for country pairs where absolute bilateral exports are larger. To see the latter, compare the values for "Domestic large" versus "Domestic small", pertaining to domestic sales and brackets of their relative magnitude in the data. Or compare exports in the group "Large-large" – or even "Large-small" and "Small-large" – with ones in the group "Small-small".

To conclude, the BvOLS-approach performs poorly in counterfactual experiments which are related to trade-cost changes relative to the observed benchmark state, because its approximation point is distant and because it involves an inappropriate linearization at the approximation point using sales and expenditure shares which are not associated with this point.

Table 4: The bias of predicted general-equilibrium-consistent counterfactual effects for various approaches

	AvW	Bias BvOLS		Bias EP_1	
5% reduction	mean	mean	sd	mean	sd
Domestic small	−0.039	−0.020	0.009	−0.004	0.002
Domestic large	−0.016	−0.038	0.007	0.001	0.002
Small-small	0.012	−0.020	0.008	−0.004	0.002
Large-large	0.035	−0.038	0.014	0.001	0.002
Small-large	0.023	−0.029	0.012	−0.002	0.002
Large-small	0.024	−0.029	0.011	−0.001	0.002
Total	0.022	−0.029	0.013	−0.002	0.002
25% reduction					
Domestic small	−0.229	−0.101	0.049	−0.011	0.014
Domestic large	−0.098	−0.207	0.042	0.010	0.012
Small-small	0.059	−0.101	0.042	−0.011	0.010
Large-large	0.189	−0.207	0.072	0.010	0.015
Small-large	0.122	−0.152	0.061	−0.002	0.009
Large-small	0.127	−0.156	0.057	0.001	0.015
Total	0.117	−0.154	0.070	−0.001	0.015

Notes: AvW refers to the true counterfactual change obtained by solving the nonlinear Anderson and van Wincoop (2003) general-equilibrium model. There may be some bias with that model even, as it has to be estimated and as there is a residual term that generates a gap between the model and the data. BvOLS uses observed sales and expenditure shares and uniform trade costs as the approximation point. EP_1 refers to the within estimates with true bilateral trade flows as weights.

5 Conclusions

Structural-quantitative models of aggregate bilateral trade at the country-pair (or country-pair-sector country-pair-product) level exhibit a structure which is nonlinear in the parameters.

Three leading approaches to estimate parameters while respecting general-equilibrium constraints exist: (i) one which controls for general-equilibrium effects through the inclusion of country-fixed effects (see Eaton and Kortum, 2002); (ii) one which estimates the parameters by conditioning on iteratively solved equilibrium constraints in a nonlinear estimation model (see Anderson and van Wincoop, 2003); and (iii) a linearization of the model, dubbed Bonus-vetus-OLS (BvOLS or Good Old OLS; see Baier and Bergstrand 2009a) – which intends to combine the merits of (i) and (ii) by conditioning on linearized versions of the nonlinear equilibrium-constraint terms in OLS estimation without fixed effects. The latter enjoys popularity among practitioners not only in international economics but beyond.

This paper shows that BvOLS suffers from a bias in the linearization (or approximation) point, which leads to a parameter-estimation bias as well as a bias in the quantification of the effects of trade-cost changes. We quantify the overall bias and decompose it. Moreover, we propose an alternative linearization approach which serves as a remedy and approximates the model at data-supported points.

Appendix

A Proofs of the Propositions and Corollaries

A.1 Approximating the System of Multilateral Resistances, Proof of Proposition 1

As shown in Section 2.2, the system of multilateral resistances can be written as an implicit function

$$\mathbf{F}(\mathbf{t}) = \begin{bmatrix} \sum_{i=1}^{N} e^{\alpha t_{ij}+\mu_i(\mathbf{t})+m_j(\mathbf{t})} - \theta_j(\mathbf{t}),\ j=2,...,N \\ ... \\ \sum_{j=1}^{N} e^{\alpha t_{ij}+\mu_i(\mathbf{t})+m_j(\mathbf{t})} - \kappa_i(\mathbf{t}),\ i=1,...,N \end{bmatrix} = \mathbf{0}.$$

Remember the normalization $m_1(\mathbf{t}) = 0$ and that the system of trade resistances comprises $2N-1$ interdependent equations and variables. Also recall that index a pertains to the point of approximation that replicates the baseline equilibrium at $t_{ij,a}$ and that trade costs are parameterized by a single explanatory variable to simplify notation.

Abbreviating $\mu_{i,a} \equiv \mu_i(\mathbf{t}_a)$ and $m_{j,a} \equiv m_j(\mathbf{t}_a)$ and ignoring the remainder linearization error, the system of multilateral resistance terms can then be linearly approximated as

$$\mathbf{F}(\mathbf{t}) \approx \mathbf{F}(\mathbf{t}_a) + \begin{cases} \theta_{j,a}(m_j - m_{j,a}) & j=2,...,N \\ +\sum_{i=1}^{N} \widetilde{X}_{ij,a}(\mu_i - \mu_{i,a}) + \alpha\sum_{i=1}^{N} \widetilde{X}_{ij,a}(t_{ij} - t_{ij,a}) & \\ & \\ \sum_{j=2}^{N} \widetilde{X}_{ij,a}(m_j - m_{j,a}) & i=1,...,N \\ +\kappa_{i,a}(\mu_i - \mu_{i,a}) + \alpha\sum_{j=1}^{N} \widetilde{X}_{ij,a}(t_{ij} - t_{ij,a}). & \end{cases}$$

Note we insert

$$\begin{aligned} \theta_{j,a} &= \sum_{i=1}^{N} e^{\alpha t_{ij}+\mu_{i,a}+m_{j,a}},\ j=2,...,N \\ \kappa_{i,a} &= \sum_{j=1}^{N} e^{\alpha t_{ij}+\mu_{i,a}+m_{j,a}},\ i=1,...,N. \end{aligned}$$

This allows us to define the $(2N-1 \times 2N-1)$ matrix of first derivatives. For this, we collect binary exporter dummies in $\mathbf{D}_x$ and importer dummies in $\mathbf{D}_m$ (skipping importer country 1), and the two together in the $N^2 \times (2N-1)$ design matrix

$\mathbf{D} = [\mathbf{D}_m, \mathbf{D}_x]$. A constant is absent from this model to avoid multicollinearity. It is straightforward to show that

$$\left[\begin{array}{c} \frac{\partial \mathbf{F}(\mathbf{t})}{\partial \mathbf{m}'} \\ \frac{\partial \mathbf{F}(\mathbf{t})}{\partial \boldsymbol{\mu}'} \end{array}\right]_{\mathbf{t}=\mathbf{t}_a} = \mathbf{D}'\mathbf{G}(\mathbf{t}_a)\mathbf{D},$$

where $\mathbf{G}(\mathbf{t})$=$diag(\widetilde{X}_{ij}(\mathbf{t}))$ is an $N^2 \times N^2$ matrix and we abbreviate $\mathbf{G}_a \equiv \mathbf{G}(\mathbf{t}_a)$.

Defining $\Delta\mathbf{t}_a$= $\mathbf{t} - \mathbf{t}_a$, the linear approximation of the system of $2N - 1$ multilateral resistance equations can thus be compactly written in matrix form as

$$\underbrace{\mathbf{F}(\mathbf{t})}_{0} = \underbrace{\mathbf{F}(\mathbf{t}_a)}_{0} + \alpha\mathbf{D}'\mathbf{G}_a\Delta\mathbf{t}_a + \mathbf{D}'\mathbf{G}_a\mathbf{D}\left[\begin{array}{c} \Delta\mathbf{m}(\mathbf{t}, \mathbf{t}_a) \\ \Delta\boldsymbol{\mu}(\mathbf{t}, \mathbf{t}_a) \end{array}\right] + \mathbf{r}(\mathbf{t}, \mathbf{t}_a),$$

where $\mathbf{r}(\mathbf{t}, \mathbf{t}_a)$ denotes the approximation error. Since $\mathbf{D}'\mathbf{G}(\mathbf{t}_a)\mathbf{D}$ has full rank and is invertible, it follows that

$$\left[\begin{array}{c} \Delta\mathbf{m}(\mathbf{t}, \mathbf{t}_a) \\ \Delta\boldsymbol{\mu}(\mathbf{t}, \mathbf{t}_a) \end{array}\right] = -\left(\mathbf{D}'\mathbf{G}_a\mathbf{D}\right)^{-1}\left(\alpha\mathbf{D}'\mathbf{G}_a\Delta\mathbf{t}_a + \mathbf{r}(\mathbf{t}, \mathbf{t}_a)\right).$$

In vector form, with the vector of log normalized trade flows $\widetilde{\mathbf{x}}(\mathbf{t})$ containing elements $\widetilde{x}_{ij}(\mathbf{t}) = \log\left(\widetilde{X}_{ij}(\mathbf{t})\right)$, the model can be compactly written as

$$\widetilde{\mathbf{x}}(\mathbf{t}) = \alpha\mathbf{t} + \mathbf{D}\left[\begin{array}{c} \mathbf{m}(\mathbf{t}) \\ \boldsymbol{\mu}(\boldsymbol{t}) \end{array}\right],$$

and its linear approximation at $\mathbf{t}_a$ is given as

$$\widetilde{\mathbf{x}}(\mathbf{t}) = \widetilde{\mathbf{x}}_a + \alpha\Delta\mathbf{t}_a - \mathbf{D}\left(\mathbf{D}'\mathbf{G}_a\mathbf{D}\right)^{-1}\left(\alpha\mathbf{D}'\mathbf{G}_a\Delta\mathbf{t}_a + \mathbf{r}(\mathbf{t}, \mathbf{t}_a)\right).$$

We decompose

$$\widetilde{\mathbf{x}}(\mathbf{t}) = \widetilde{\mathbf{x}}_L(\mathbf{t}, \mathbf{t}_a) + \widetilde{\mathbf{r}}(\mathbf{t}, \mathbf{t}_a),$$

where

$$\widetilde{\mathbf{x}}_L(\mathbf{t}, \mathbf{t}_a) = \widetilde{\mathbf{x}}_a + \alpha\left(\underbrace{\mathbf{I} - \mathbf{D}\left(\mathbf{D}'\mathbf{G}_a\mathbf{D}\right)^{-1}\mathbf{D}'\mathbf{G}_a}_{\mathbf{Q}_a = \mathbf{I} - \mathbf{P}_a}\right)\Delta\mathbf{t}_a = \widetilde{\mathbf{x}}_a + \alpha\mathbf{Q}_a\Delta\mathbf{t}_a$$

and

$$
\begin{aligned}
\widetilde{\mathbf{r}}(\mathbf{t},\mathbf{t}_a) &= \widetilde{\mathbf{x}}(\mathbf{t})-\widetilde{\mathbf{x}}_L(\mathbf{t},\mathbf{t}_a)\\
&= \alpha\mathbf{t}+\mathbf{D}\begin{bmatrix}\mathbf{m}(\mathbf{t})\\ \boldsymbol{\mu}(\boldsymbol{t})\end{bmatrix}-\underbrace{\alpha\mathbf{t}_a-\mathbf{D}\begin{bmatrix}\mathbf{m}(\mathbf{t}_a)\\ \boldsymbol{\mu}(\mathbf{t}_a)\end{bmatrix}}_{\widetilde{\mathbf{x}}_a}-\alpha\Delta\mathbf{t}_a+\alpha\underbrace{\mathbf{D}\left(\mathbf{D}'\mathbf{G}_a\mathbf{D}\right)^{-1}\mathbf{D}'\mathbf{G}_a}_{\mathbf{P}_a}\Delta\mathbf{t}_a\\
&= \alpha\mathbf{P}_a\Delta\mathbf{t}_a+\mathbf{D}\begin{bmatrix}\Delta\mathbf{m}(\mathbf{t},\mathbf{t}_a)\\ \Delta\boldsymbol{\mu}(\mathbf{t},\mathbf{t}_a)\end{bmatrix}\\
&= \mathbf{D}\Big(\alpha\left(\mathbf{D}'\mathbf{G}_a\mathbf{D}\right)^{-1}\mathbf{D}'\mathbf{G}_a\Delta\mathbf{t}_a-\left(\mathbf{D}'\mathbf{G}_a\mathbf{D}\right)^{-1}\left(\alpha\mathbf{D}'\mathbf{G}_a\Delta\mathbf{t}_a+\mathbf{r}(\mathbf{t},\mathbf{t}_a)\right)\Big)\\
&= -\mathbf{D}\left(\mathbf{D}'\mathbf{G}_a\mathbf{D}\right)^{-1}\mathbf{r}(\mathbf{t},\mathbf{t}_a),
\end{aligned}
$$

using

$$
\begin{bmatrix}\Delta\mathbf{m}(\mathbf{t},\mathbf{t}_a)\\ \Delta\boldsymbol{\mu}(\mathbf{t},\mathbf{t}_a)\end{bmatrix}=-\left(\mathbf{D}'\mathbf{G}_a\mathbf{D}\right)^{-1}\left(\alpha\mathbf{D}'\mathbf{G}_a\Delta\mathbf{t}_a+\mathbf{r}(\mathbf{t},\mathbf{t}_a)\right).
$$

A.2 The BvOLS-Approximation and its Approximation Error, Proof of Corollary 1

The BB-linearization applies Proposition 1 setting $\mathbf{t}_{BB}=\mathbf{t}\iota_{N^2}$ (i.e., $a=BB$) and using $\mathbf{G}_{BB}=diag(\kappa_i(\mathbf{t}_{BB})\theta_j(\mathbf{t}_{BB}))$. The $BvOLS$-approximation uses the observed sales and expenditure shares and is given as

$$
\widetilde{\mathbf{x}}_{L,BvOLS}(\mathbf{t},\mathbf{t}_{BB})=\widetilde{\mathbf{x}}_{BvOLS}+\alpha\mathbf{Q}_{BvOLS}\Delta\mathbf{t}_{BB}.
$$

Using

$$
\begin{aligned}
\widetilde{\mathbf{x}}(\mathbf{t}) &= \widetilde{\mathbf{x}}_{BB}+\alpha\mathbf{Q}_{BB}\Delta\mathbf{t}_{BB}+\widetilde{\mathbf{r}}(\mathbf{t},\mathbf{t}_{BB})\\
\widetilde{\mathbf{r}}(\mathbf{t},\mathbf{t}_{BB}) &= -\mathbf{D}\left(\mathbf{D}'\mathbf{G}_a\mathbf{D}\right)^{-1}\mathbf{r}(\mathbf{t},\mathbf{t}_{BB})
\end{aligned}
$$

yields the approximation error

$$
\begin{aligned}
\widetilde{\mathbf{x}}(\mathbf{t})-\widetilde{\mathbf{x}}_{L,BvOLS}(\mathbf{t},\mathbf{t}_{BB}) &= \widetilde{\mathbf{x}}_{BB}+\alpha\mathbf{Q}_{BB}\Delta\mathbf{t}_{BB}+\widetilde{\mathbf{r}}(\mathbf{t},\mathbf{t}_{BB})-\widetilde{\mathbf{x}}_{BvOLS}+\alpha\mathbf{Q}_{BvOLS}\Delta\mathbf{t}_{BB}\\
&= \left(\widetilde{\mathbf{x}}_{BB}-\widetilde{\mathbf{x}}_{BvOLS}\right)+\alpha\left(\mathbf{Q}_{BB}-\mathbf{Q}_{BvOLS}\right)\Delta\mathbf{t}_{BB}+\widetilde{\mathbf{r}}(\mathbf{t},\mathbf{t}_{BB}).
\end{aligned}
$$

A.3 BvOLS-Parameter Estimates, Bias and Predictions. Proof of Proposition 2

The true econometric model gravity model can be compactly written as

$$
\widetilde{\mathbf{x}}(\mathbf{t})=\alpha\mathbf{t}+\mathbf{D}\begin{bmatrix}\mathbf{m}(\mathbf{t})\\ \boldsymbol{\mu}(\mathbf{t})\end{bmatrix}+\boldsymbol{\varepsilon},\ \text{s.t. } \mathbf{F}(\mathbf{t})=\mathbf{0},
$$

where ε denotes the disturbances. At the approximation point $\mathbf{t}_{BB}$, the BvOLS-model in logs reads

$$\Delta\widetilde{\mathbf{x}}_{BvOLS} = \widetilde{\mathbf{x}}(\mathbf{t}) - \widetilde{\mathbf{x}}_{BvOLS}(\mathbf{t}_{BB}) = \alpha\mathbf{Q}_{BvOLS}\Delta\mathbf{t}_{BB} + \widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t},\mathbf{t}_{BB}) + \boldsymbol{\varepsilon}, \qquad (9)$$

where $\widetilde{\mathbf{x}}_{BvOLS}$ has typical element $\ln\kappa_i + \ln\theta_j$ at the point of approximation and $\Delta\mathbf{t}_{BB}$ has typical element $t_{ij} - t$ for all i and j. $\mathbf{Q}_{BvOLS} = \mathbf{I} - (\mathbf{D}'\mathbf{G}_{BvOLS}\mathbf{D})^{-1}\mathbf{D}'\mathbf{G}_{BvOLS}$ with $\mathbf{G}_{BvOLS} = diag(\kappa_i\theta_j)$. Applying OLS, the BvOLS estimate of α is easily derived as

$$\widehat{\alpha}_{BvOLS} = (\Delta\mathbf{t}'_{BB}\mathbf{Q}'_{BvOLS}\mathbf{Q}_{BvOLS}\Delta\mathbf{t}_{BB})^{-1}\Delta\mathbf{t}'_{BB}\mathbf{Q}'_{BvOLS}\Delta\widetilde{\mathbf{x}}_{BvOLS}.$$

Inserting the true data-generating process (9) and defining $\mathbf{H}_{BvOLS} = \Delta\mathbf{t}'_{BB}\mathbf{Q}'_{BvOLS}\mathbf{Q}_{BvOLS}\Delta\mathbf{t}_{BB}$ yields

$$\begin{aligned} \widehat{\alpha}_{BvOLS} &= \mathbf{H}^{-1}_{BvOLS}\Delta\mathbf{t}'_{BB}\mathbf{Q}'_{BvOLS}\Big[\underbrace{\alpha\mathbf{Q}_{BvOLS}\Delta\mathbf{t}_{BB} + \widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t},\mathbf{t}_{BB}) + \boldsymbol{\varepsilon}}_{\text{True model for }\Delta\widetilde{\mathbf{x}}}\Big] \\ &= \alpha + \mathbf{H}^{-1}_{BvOLS}\Delta\mathbf{t}'_{BB}\mathbf{Q}'_{BvOLS}\left(\widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t},\mathbf{t}_{BB}) + \boldsymbol{\varepsilon}\right). \end{aligned}$$

As shown in Corollary 1, the BvOLS-approximation error is given as

$$\widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t},\mathbf{t}_{BB}) = \widetilde{\mathbf{x}}_{BB} - \widetilde{\mathbf{x}}_{BvOLS} + \alpha(\mathbf{Q}_{BB} - \mathbf{Q}_{BvOLS})\Delta\mathbf{t}_{BB} + \widetilde{\mathbf{r}}(\mathbf{t},\mathbf{t}_{BB}).$$

Next observe that for any approximation points a and a' with asymmetric sales and expenditure shares it holds that

$$\begin{aligned} \mathbf{P}_a\mathbf{P}_{a'} &= \mathbf{D}\left(\mathbf{D}'\mathbf{G}_a\mathbf{D}\right)^{-1}\mathbf{D}'\mathbf{G}_a\mathbf{D}\left(\mathbf{D}'\mathbf{G}_{a'}\mathbf{D}\right)^{-1}\mathbf{D}'\mathbf{G}_{a'} = \mathbf{P}_{a'} \\ \mathbf{P}'_a\mathbf{P}_{a'} &= \mathbf{G}_a\mathbf{D}\left(\mathbf{D}'\mathbf{G}_a\mathbf{D}\right)^{-1}\mathbf{D}'\mathbf{D}\left(\mathbf{D}'\mathbf{G}_{a'}\mathbf{D}\right)^{-1}\mathbf{D}'\mathbf{G}_{a'} \neq \mathbf{P}_{a'} \\ \mathbf{Q}'_a\mathbf{Q}_{a'} &= (\mathbf{I}_{N^2} - \mathbf{P}'_a)(\mathbf{I}_{N^2} - \mathbf{P}_{a'}) = \mathbf{I}_{N^2} - \mathbf{P}'_a - \mathbf{P}_{a'} + \mathbf{P}'_a\mathbf{P}_{a'} \neq \mathbf{Q}_a \\ \mathbf{Q}'_a\mathbf{D} &= \mathbf{D} - \mathbf{G}_a\mathbf{D}\left(\mathbf{D}'\mathbf{G}_a\mathbf{D}\right)^{-1}\mathbf{D}'\mathbf{D} \neq \mathbf{0}. \end{aligned}$$

It follows that the BvOLS estimate of α exhibits a bias, i.e., $\mathbf{Q}'_{BvOLS}\widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t},\mathbf{t}_{BB}) \neq \mathbf{0}$, since

$$\begin{aligned} \mathbf{Q}_{BvOLS}\mathbf{D} &\neq \mathbf{0}, \\ \mathbf{Q}'_{BvOLS}\mathbf{Q}_{BB} &\neq \mathbf{0}, \\ \mathbf{Q}'_{BvOLS}\mathbf{Q}_{BvOLS} &\neq \mathbf{0}. \end{aligned}$$

Hence, $\mathbf{Q}'_{Bvols}(\mathbf{Q}_{Bvols} - \mathbf{Q}_{BB}) \neq \mathbf{0}$. Moreover, although $\widetilde{\mathbf{x}}_{BB} - \widetilde{\mathbf{x}}_{BvOLS}$ lacks bilateral variation, it does not disappear as $\mathbf{Q}'_{BvOLS}\mathbf{D} \neq \mathbf{0}$.

Taking expectations and assuming exogeneity of trade barriers allows to calculate the bias of $\widehat{\alpha}_{BvOLS}$ as

$$Bias_{BvOLS} \equiv E[\widehat{\alpha}_{BvOLS} - \alpha|\mathbf{t},\boldsymbol{\kappa},\boldsymbol{\theta}] = \alpha\mathbf{H}^{-1}_{BvOLS}\Delta\mathbf{t}'_{BB}\mathbf{Q}'_{BvOLS}\widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t},\mathbf{t}_{BB}).$$

Assuming that disturbances are independent but heteroskedastic with diagonal variance-covariance matrix $\mathbf{\Omega}_{\varepsilon}$, it follows that

$$\begin{aligned}
& E[(\widehat{\alpha}_{BvOLS}-\alpha-Bias_{BvOLS})(\widehat{\alpha}_{BvOLS}-\alpha-Bias_{BvOLS})'|\mathbf{t},\boldsymbol{\kappa},\boldsymbol{\theta}] \\
= \; & \mathbf{H}_{BvOLS}^{-1}\Delta\mathbf{t}'_{BB}\mathbf{Q}'_{BvOLS}\mathbf{\Omega}_{\varepsilon}\mathbf{Q}_{BvOLS}\Delta\mathbf{t}_{BB}\mathbf{H}_{BvOLS}^{-1}.
\end{aligned}$$

The bias disappears if the BvOLS estimator is unweighted. Using the subscript $a = w$ for this case and setting $\mathbf{G}_w = \mathbf{I}_{N^2}$ yields symmetric and idempotent projection matrices $\mathbf{P}_w = \mathbf{D}\left(\mathbf{D}'\mathbf{D}\right)^{-1}\mathbf{D}'$, $\mathbf{Q}_w = \mathbf{I}_{N^2} - \mathbf{P}_w$ and $\mathbf{Q}_w\mathbf{D}= \mathbf{0}$.

$$\widehat{\alpha}_w = \left(\Delta\mathbf{t}'_{BB}\mathbf{Q}_w\Delta\mathbf{t}_{BB}\right)^{-1}\Delta\mathbf{t}'_{BB}\mathbf{Q}_w\left(\alpha\mathbf{Q}_{BB}\Delta\mathbf{t}_{BB}+\widetilde{\mathbf{r}}_{BB}(\mathbf{t},\mathbf{t}_{BB})+\boldsymbol{\varepsilon}\right).$$

The bias disappears, because $\widetilde{\mathbf{r}}(\mathbf{t},\mathbf{t}_{BB})$ exhibits unilateral variation only and $\mathbf{Q}_w\mathbf{D} = \mathbf{0}$. Further, $\mathbf{Q}_w\mathbf{Q}_{BB} = \mathbf{Q}_w$, since

$$\begin{aligned}
\mathbf{P}_w\mathbf{P}_{BB} &= \mathbf{D}\left(\mathbf{D}'\mathbf{D}\right)^{-1}\mathbf{D}'\mathbf{D}\left(\mathbf{D}'\mathbf{G}_{BB}\mathbf{D}\right)^{-1}\mathbf{D}'\mathbf{G}_{BB} = \mathbf{P}_{BB}, \\
\mathbf{Q}_w\mathbf{Q}_{BB} &= \mathbf{I}_{N^2} - \mathbf{P}_w - \mathbf{P}_{BB} + \mathbf{P}_w\mathbf{P}_{BB} = \mathbf{Q}_w.
\end{aligned}$$

Then,

$$\widehat{\alpha}_w - \alpha = \left(\Delta\mathbf{t}'_{BB}\mathbf{Q}_w\Delta\mathbf{t}_{BB}\right)^{-1}\Delta\mathbf{t}'_{BB}\mathbf{Q}_w\boldsymbol{\varepsilon}.$$

and $E[\widehat{\alpha}_w - \alpha|\mathbf{t},\boldsymbol{\kappa},\boldsymbol{\theta}] = \mathbf{0}$. Hence, one obtains the standard within-estimator of a linear two-way panel model.

A.4 Counterfactual Changes. Proof of Corollary 2

Consider a counterfactual change in the trade frictions of $\mathbf{t}^c - \mathbf{t}$, and assume that both the base and counterfactual are predicted with BvOLS. Ignoring the associated change in sales and expenditure shares, the structural gravity model implies that counterfactual changes are given as

$$\begin{aligned}
E[\widetilde{\mathbf{x}}^c - \widetilde{\mathbf{x}}|\mathbf{t},\mathbf{t}^c,\boldsymbol{\kappa},\boldsymbol{\theta}] &= \mathbf{Q}_{BvOLS}\Delta\mathbf{t}^c\alpha - \mathbf{Q}_{BvOLS}\Delta\mathbf{t}\alpha \\
&\quad + \widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t}^c,\mathbf{t}_{BB}) - \widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t},\mathbf{t}_{BB}) \\
&= \mathbf{Q}_{BvOLS}\left(\mathbf{t}^c - \mathbf{t}\right)\alpha + \widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t}^c,\mathbf{t}_{BB}) - \widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t},\mathbf{t}_{BB}),
\end{aligned}$$

where $\widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t}^c,\mathbf{t}_{BB})$ and $\widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t},\mathbf{t}_{BB})$ denote the BvOLS-linearization errors as defined in Corollary 1.

The BvOLS-approach ignores the remainder error and proceeds with the estimate

$$\widehat{\Delta\widetilde{\mathbf{x}}^c}_{L,BvOLS} - \widehat{\Delta\widetilde{\mathbf{x}}}_{L,BvOLS} = \mathbf{Q}_{BvOLS}\left(\mathbf{t}^c - \mathbf{t}\right)\widehat{\alpha}_{BvOLS}.$$

The estimated log counterfactual change under BvOLS is thus obtained as

$$E[\widehat{\Delta\widetilde{\mathbf{x}}^c}_{BvOLS} - \widehat{\Delta\widetilde{\mathbf{x}}}_{BvOLS}|\mathbf{t},\mathbf{t}^c,\boldsymbol{\kappa},\boldsymbol{\theta}] = \mathbf{Q}_{BvOLS}\left(\mathbf{t}^c - \mathbf{t}\right)\left(\alpha + Bias_{BvOLS}\right),$$

considering that both $\widehat{\widetilde{\mathbf{x}}}_{BB}$ and $\widehat{\widetilde{\mathbf{x}}}^c_{BB}$ are approximated at the same point $\mathbf{t}_{BB} = t\boldsymbol{\iota}_{N^2}$. The expected prediction error thus amounts to

$$\begin{aligned} & E\left[\left(\widehat{\widetilde{\mathbf{x}}}^c_{BvOLS} - \widehat{\widetilde{\mathbf{x}}}_{BvOLS}\right) - (\widetilde{\mathbf{x}}^c - \widetilde{\mathbf{x}})\,|\mathbf{t}, \boldsymbol{\kappa}, \boldsymbol{\theta}\right] \\ = \;& \mathbf{Q}_{BvOLS}\,(\mathbf{t}^c - \mathbf{t})\,Bias_{BvOLS} - \widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t}^c, \mathbf{t}_{BB}) + \widetilde{\mathbf{r}}_{BvOLS}(\mathbf{t}, \mathbf{t}_{BB})). \end{aligned}$$

An alternative is to predict both base and counterfactual, the latter with a linearization at $\mathbf{t}$, as observed in the base (with subscript EP_1) using the unbiased within estimator:

$$\widehat{\widetilde{\mathbf{x}}}^c_{EP_1} - \widehat{\widetilde{\mathbf{x}}}_{EP_1} = \mathbf{Q}_{EP_1}(\mathbf{t}^c - \mathbf{t})\widehat{\alpha}_w,$$

where $\mathbf{Q}_{EP_1} = \mathbf{I}_{N^2} - \mathbf{D}\left(\mathbf{D}'\mathbf{G}_{EP_1}\mathbf{D}\right)^{-1}\mathbf{D}'\mathbf{G}_{EP_1}$ and $\mathbf{G}_{EP_1} = diag(\widehat{\widetilde{X}}_{ij,EP_1}(\mathbf{t}))$. Since the baseline prediction is free of bias, $\widetilde{\mathbf{r}}_{EP_1}(\mathbf{t}, \mathbf{t}) = 0$, the bias of the predicted counterfactual change with the EP_1 linearization is given as

$$\begin{aligned} & E\left[\left(\widehat{\widetilde{\mathbf{x}}}^c_{EP_1} - \widehat{\widetilde{\mathbf{x}}}_{EP_1}\right) - (\widetilde{\mathbf{x}}^c - \widetilde{\mathbf{x}})\,|\mathbf{t}, \mathbf{t}^c, \boldsymbol{\kappa}, \boldsymbol{\theta}\right] \\ = \;& E\left[\widehat{\alpha}_w\mathbf{Q}_{EP_1}(\mathbf{t}^c - \mathbf{t})|\mathbf{t}, \mathbf{t}^c, \boldsymbol{\kappa}, \boldsymbol{\theta}\right] - \alpha\mathbf{Q}_{EP_1}(\mathbf{t}^c - \mathbf{t}) - \widetilde{\mathbf{r}}_{EP_1}(\mathbf{t}^c, \mathbf{t}) \\ = \;& -\widetilde{\mathbf{r}}_{EP_1}(\mathbf{t}^c, \mathbf{t}). \end{aligned}$$

A.5 Endogenous Sales and Expenditure Shares in the $\mathbf{EP_2}$ -Model

The proposed EP_1- as well as the BvOLS-approximation in the main text ignore that the sales and expenditure shares adjust endogenously in response to changes in trade barriers. To account for the latter, observe that in an endowment economy as assumed in Section 2, the ratio of sales between any counterfactual value and the observed base value with index a behaves according to

$$\frac{Y_i}{Y_{i,a}} = \frac{p_i A_i}{p_{i,a} A_i} = e^{\frac{\mu_i - \mu_{i,a}}{\alpha}},$$

where A_i denotes the endowment of economy i. Remember, $\alpha = 1 - \sigma$ is the trade elasticity. This result uses $\mu_i = \ln\left(\kappa_i \Pi_i^{-\alpha}\right) = \alpha \ln p_i$ and we have

$$\begin{aligned} \kappa_i &= \frac{Y_i}{\sum_{k=1}^N Y_k} = \frac{\frac{e^{\frac{\mu_i - \mu_{i,a}}{\alpha}} Y_{i,a}}{\sum_{k=1}^N Y_{k,a}}}{\frac{\sum_{k=1}^N e^{\frac{\mu_k - \mu_{k,a}}{\alpha}} Y_{k,a}}{\sum_{l=1}^N Y_{l,a}}} = \frac{e^{\frac{\mu_i - \mu_{i,a}}{\alpha}} \kappa_{i,a}}{\sum_{k=1}^N e^{\frac{\mu_k - \mu_{k,a}}{\alpha}} \kappa_{k,a}} \\ \theta_j &= b_j \kappa_j. \end{aligned}$$

$b_j \equiv \theta_j/\kappa_j$ denotes the fixed trade imbalance of country j. κ_i and θ_j denote the sales and expenditure shares, respectively. Deviating from the main text, for convenience we normalize $\mu_N(\mathbf{t}) = 0$, rather than $m_1(\mathbf{t}) = 0$. In addition, we use the $(2N-1) \times N$ matrix $\mathbf{B}$, which is implicitly defined so that $[\boldsymbol{\theta}(\boldsymbol{\mu})', \boldsymbol{\kappa}(\boldsymbol{\mu})']' = \mathbf{B}\boldsymbol{\kappa}(\boldsymbol{\mu})$. In matrix form, we can then write the system of multilateral resistances compactly as

$$\mathbf{F}(\mathbf{t}) = \mathbf{D}'\widetilde{\mathbf{X}}(\mathbf{t}) - \mathbf{B}\boldsymbol{\kappa}(\boldsymbol{\mu}(\mathbf{t})) = \mathbf{0},$$

where $\widetilde{\mathbf{X}}(\mathbf{t})$ denotes the vector of normalized trade flows in levels. Furthermore, we define $h_{i,a} = \frac{\kappa_{i,a}}{\sum_{k=1}^{N} \kappa_{k,a}}$, $\mathbf{h}_a = (h_{1,a}, ..., h_{N,a})'$, $\mathbf{h}_{-1,a} = (h_{1,a}, ..., h_{N-1,a})'$. Approximation at $m_i = m_{i,a}$ and $\mu_i = \mu_{i,a}$ obtains

$$\begin{bmatrix} \boldsymbol{\theta}(\boldsymbol{\mu}) \\ \boldsymbol{\kappa}(\boldsymbol{\mu}) \end{bmatrix} = \mathbf{B}\boldsymbol{\kappa}(\boldsymbol{\mu}) = \mathbf{B}\boldsymbol{\kappa}(\boldsymbol{\mu}_a) + \mathbf{H}_a \begin{bmatrix} \mathbf{m} - \mathbf{m}_a \\ \boldsymbol{\mu}_{-1} - \boldsymbol{\mu}_{-1,a} \end{bmatrix} + \mathbf{r}_\kappa(\Delta\boldsymbol{\mu}_a),$$

where

$$\mathbf{H}_{a,2N-1\times 2N-1} \equiv \left. \frac{\partial \mathbf{B}\boldsymbol{\kappa}(\boldsymbol{\mu}_{-1})}{\partial \left[\mathbf{m}', \boldsymbol{\mu}'_{-1}\right]} \right|_{\mathbf{m}=\mathbf{m}_a, \boldsymbol{\mu}_{-1}=\boldsymbol{\mu}_{-1,a}}$$

$$= \begin{bmatrix} \mathbf{0}_{(2N-1)\times N} & \frac{1}{\alpha}\mathbf{B}_{(2N-1)\times N} \left(diag(\mathbf{h}_a) - \mathbf{h}_a \mathbf{h}'_{-1,a} \right)_{N\times(N-1)} \end{bmatrix},$$

$\boldsymbol{\mu}'_{-1} = [\mu_1, ..., \mu_{N-1}]$ and $\mathbf{r}_\kappa(\mathbf{t}, \mathbf{t}_a)$ denotes the remainder approximation error. The linear approximation of $\mathbf{F}(\mathbf{t})$ at $\mathbf{t}_a$ is given as

$$\underbrace{\mathbf{F}(\mathbf{t})}_{0} = \underbrace{\mathbf{F}(\mathbf{t}_a)}_{0} + \alpha\mathbf{D}'\mathbf{G}_a\Delta\mathbf{t}_a + (\mathbf{D}'\mathbf{G}_a\mathbf{D} - \mathbf{H}_a) \begin{bmatrix} \Delta\mathbf{m}(\mathbf{t}, \mathbf{t}_a) \\ \Delta\boldsymbol{\mu}_{-1}(\mathbf{t}, \mathbf{t}_a) \end{bmatrix} + \mathbf{r}(\mathbf{t}, \mathbf{t}_a) - \mathbf{r}_\kappa(\Delta\boldsymbol{\mu}_a).$$

Hence, the approximation of the system of multilateral resistance equations with endogenous adjustment of sales and expenditure shares and its linearization error can be written as

$$\begin{aligned} \widetilde{\mathbf{x}}_{L,GE}(\mathbf{t}_a) &= \widetilde{\mathbf{x}}_a + \alpha \left(\mathbf{I} - \mathbf{D} \left(\mathbf{D}'\mathbf{G}_a\mathbf{D} - \mathbf{H}_a \right)^{-1} \mathbf{D}'\mathbf{G}_a \right) \Delta\mathbf{t}_a \\ \widetilde{\mathbf{r}}_{GE}(\mathbf{t}, \mathbf{t}_a) &= -\alpha\mathbf{D} \left(\mathbf{D}'\mathbf{G}_a\mathbf{D} - \mathbf{H}_a \right)^{-1} \left(\mathbf{r}(\mathbf{t}, \mathbf{t}_a) - \mathbf{r}_\kappa(\Delta\boldsymbol{\mu}_a) \right). \end{aligned}$$

B Further Monte Carlo Simulation Results

In an alternative design, the disturbances ε_{ij} were drawn from the normal distribution with mean 0 and variance $\sigma_{ij}^2 = exp(0.25d_{ij})/N^{-2}\sum_{i=1}^{N}\sum_{j=1}^{N} exp(0.25d_{ij}))$, where d_{ij} denotes log distance. The average of σ_{ij} was normalized to 0.72 to match the average standard error of the wild bootstrap disturbances.

The results are summarized in Table A.1, and they can be compared with those in Table 2 of the main text. Overall, the two sets of results support similar conclusions.

Table A1: Monte Carlo simulation results, alternative data-generating process

Bias in percent	ζ_{AvW}	ζ_{BB}	ζ_{BvOLS}	ζ_w
Border	0.07	21.60	12.04	0.06
Log distance	0.02	8.71	9.42	0.06
Contiguity	−0.40	−53.84	−39.47	−0.33
Common language	0.28	−39.50	−26.47	−0.09
RTA	0.04	19.94	19.24	−0.07
Root mean-squared error				
Border	0.16	0.49	0.30	0.18
Log distance	0.04	0.08	0.08	0.04
Contiguity	0.10	0.17	0.14	0.09
Common language	0.10	0.15	0.12	0.09
RTA	0.05	0.13	0.13	0.08

Notes: Index AvW pertains to the true Anderson and van Wincoop (2003) model. $BvOLS$ indicates the usage of observed expenditure and sales shares in forming the weighted within transformation matrix, while BB stands for the use of true shares at the approximation point. Subscript w indicates two-way (unweighted within) country-fixed-effects parameters. There are 42 countries and 1764 country-pair observations and 20,000 Monte Carlo runs.

References

Afesorgbor, Sylvanus K., 2019. The impact of economic sanctions on international trade: How do threatened sanctions compare with imposed sanctions? European Journal of Political Economy 56, 11-26.

Ahlfeldt, Gabriel, 2011. If Alonso was right: modeling accessibility and explaining the residential land gradient. Journal of Regional Science 51(2), 318-338.

Aichele, Rahel and Gabriel Felbermayr, 2015. Kyoto and carbon leakage: an empirical analysis of the carbon content of bilateral trade. Review of Economics and Statistics 97(1), 104-115.

Anderson, James and Eric van Wincoop, 2003. Gravity with gravitas: A solution to the border puzzle. American Economic Review 93(1), 170-192.

Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare, 2012. New trade models, same old gains? American Economic Review 102(1), 94-130.

Atalay, Enghin, Ali Hortaçsu, Mary Jialin Li, and Chad Syverson, 2019. How Wide Is the firm border? Quarterly Journal of Economics 134(4), 1845-1882.

Atif, Rao Muhammad, Haider Mahmood, Haiyun Liu, and Haiou Mao, 2019. Determinants and efficiency of Pakistan's chemical products' exports: An application of stochastic frontier gravity model. PLoS ONE 14(5), e0217210.

Azorín, Buendía, José Daniel, Rubén MartÃnez Alpañez, and María del Mar Sánnchez de la Vega, 2024. Efficiency in the estimation of technical coefficients and interregional multipliers the Jahn methodology versus the GRAS and Gravity-RAS methodology. Investigaciones Regionales, Journal of Regional Research 58, 179-207.

Baier, Scott L. and Jeffrey H. Bergstrand, 2009a. Bonus vetus OLS: a simple method for approximating international trade-cost effects using the gravity equation. Journal of International Economics 77(1), 77-85.

Baier, Scott L. and Jeffrey H. Bergstrand, 2009b. Estimating the effects of free trade agreements on international trade flows using matching econometrics. Journal of International Economics 77(1), 63-76.

Baier, Scott L. and Jeffrey H. Bergstrand, 2010. Approximating general equilibrium impacts of trade liberalizations using the gravity equation in Peter A. G. van Bergeijk and Steven Brakman ed. The Gravity Model in International Trade: Advances and Applications, Cambridge University Press.

Baier, Scott L. and Samuel Standaert, 2020. Gravity models and empirical trade. Oxford Research Encyclopedia of Economics and Finance, Oxford University Press, 1-48.

Baltagi, Badi H., 2021. Econometric Analysis of Panel Data, 6th Edition. Springer Nature, Switzerland.

Barbero, Javier, Giovanni Mandras, Ernesto Rodríguez-Crespo, E., and Andrés Rodríguez-Pose, (2021). Quality of government and regional trade: evidence from European Union regions. Regional Studies 55(7), 1240-1251.

Behar, Alberto and Benjamin D. Nelson, 2014. Trade flows, multilateral resistance, and firm heterogeneity. Review of Economics and Statistics 96(3), 538-549.

Behar, Alberto, Philip Manners and Benjamin D. Nelson, 2013. Exports and international logistics. Oxford Bulletin of Economics and Statistics 75(6), 855-886.

Berger, Daniel, William Easterly, Nathan Nunn, and Shanker Satyanath, 2013. Commercial imperialism? political influence, and trade during the Cold War. American Economic Review 103(2), 863-896.

Bergstrand, Jeffrey H., Peter H. Egger, and Mario Larch 2013. Gravity Redux: Estimation of gravity-equation coefficients, elasticities of substitution, and general equilibrium comparative statics under asymmetric bilateral trade costs. Journal of International Economics 89(1), 110-121.

Black, William R., 1972. Interregional commodity flows: some experiments with the gravity model. Journal of Regional Science 12(1), 107-118.

Blind, Knut, Axel Mangelsdorf, and Jakob Pohlisch, 2018. The effects of cooperation in accreditation on international trade: Empirical evidence on ISO 9000 certifications. International Journal of Production Economics 198, 50-59.

Borusyak, Kirill, Xavier Jaravel, and Jann Spiess 2021. Revisiting Event Study Designs: Robust and Efficienct Estimation, Working Paper, 2021.

Bratt, Michael, 2017. Estimating the bilateral impact of nontariff measures on trade. Review of International Economics 25(5), 1105-1129.

Cai, Mattia, 2023. A calibrated gravity model of interregional trade. Spatial Economic Analysis 18(1), 89-107

Carrère, Cèline, 2006. Revisiting the effects of regional trade agreements on trade flows with proper specification of the gravity model. European Economic Review 50(2), 223-247.

Carrère, Cèline, Monika Mrázová, and Peter J. Neary, 2020. Gravity without apology: the science of elasticities, distance and trade. Economic Journal 130(628), 880-910.

Chaney, Thomas, 2008. Distorted gravity: the intensive and extensive margins of international trade. American Economic Review 98(4), 1707-1721.

Clougherty, Joseph A. and Michael Grajek, 2014. International standards and international trade: Empirical evidence from ISO 9000 diffusion. International Journal of Industrial Organization 36, 70-82.

Czaika, Mathias, and Christopher R. Parsons, 2017. The gravity of high-skilled migration policies. Demography 54(2), 603-630.

Doe Fiankor, Dela-Dem, Insa Flachsbarth, Amjad Masood, and Bernhard Brümmer, 2020. Does GlobalGAP certification promote agrifood exports? European Review of Agricultural Economics 47(1), 247-272.

Dür, Andreas, Leonardo Baccini, and Manfred Elsig, 2014. The design of inter-national trade agreements: Introducing a new dataset. Review of International Organizations 9, 353-375.

Eaton, Jonathan and Samuel Kortum, 2002. Technology, geography, and trade. Econometrica 70(5), 1741-1779.

Egger, Peter, and Mario Larch, 2008. Interdependent preferential trade agreement memberships: An empirical analysis. Journal of International Economics 76(2), 384-399.

Egger, Peter H., Gabriel Loumeau, and Nicole Loumeau, 2023. China's dazzling transport-infrastructure growth: Measurement and effects. Journal of International Economics 142, 103734.

Egger, Peter H. and Douglas R. Nelson, 2011. How bad is antidumping? Evidence from panel data. Review of Economics and Statistics 93(4), 1374-1390.

Egger, Peter H. and Kevin E. Staub (2016). GLM estimation of trade gravity models with fixed effects. Empirical Economics 50, 137-175.

Felbermayr, Gabriel and Jasmin Gröschl, 2013. Natural disasters and the effect of trade on income: A new panel IV approach. European Economic Review 58, 18-30.

Francois, Joseph and Miriam Manchin, 2013. Institutions, Infrastructure, and Trade, World Development 46, 165-175.

Gallo, Mariano, Vittorio Marzano and Fulvio Simonelli, 2012. Empirical comparison of parametric and nonparametric trade gravity models. Transportation Research Record 2269(1), 29-41.

Glick, Reuven and Alan B. Taylor, 2010. Collateral damage: trade disruption and the economic impact of war. Review of Economics and Statistics 92(1), 102-127.

Gröschl, Jasmin and Thomas Steinwachs, 2020. Do natural hazards cause international migration? CESifo Economic Studies 63(4), 445-480.

Hansen, Bruce, 2022. Econometrics, Princeton University Press, Princeton, US.

Helpman, Elhanan, Marc J. Melitz, and Yona Rubinstein, 2008. Estimating trade flows: Trading partners and trading volumes. The Quarterly Journal of Economics 123(2), 441-487.

Judd, Kenneth L., 1998. Numerical methods in economics. MIT press.

Krugman, Paul, 1980. Scale economies, product differentiation, and the pattern of trade. The American Economic Review 70(5), 950-959.

Márquez-Ramos, Laura, 2016. Port facilities, regional spillovers and exports: Empirical evidence from Spain. Papers in Regional Science 95(2), 329-351.

Mayer, Thierry and Soledad Zignago, 2011. Notes on CEPII's distances measures: the GeoDist database. *CEPII Working Paper* 2011-25.

Melitz, Marc J., 2003. The impact of trade on intra-industry reallocations and aggregate industry productivity. Econometrica 71(6), 1695-1725.

Monte Ferdinando, Stephen J. Redding, and Esteban Rossi-Hansberg, 2019. Commuting, migration, and local employment elasticities. American Economic Review 108(12), 3855-3890.

Nicita, Alessandro, and Bernard Hoekman, 2011. Trade policy, trade costs, and developing country trade. World Development 39(12), 2069-2079.

Nitsch, Volker, and Nikolaus Wolf, 2013. Tear down this wall: on the persistence of borders in trade. Canadian Journal of Economics 46(1), 154-179.

Papke, Leslie. E. and Jeffrey M. Wooldridge, 2008. Panel data methods for fractional response variables with an application to test pass rates. Journal of Econometrics 145(1-2), 121-133.

Peeters, Ludo, 2012 .Gravity and spatial structure: the case of interstate migration in Mexico. Journal of Regional Science 52(5), 819-856.

Pfaffermayr, Michael, 2020. Constrained Poisson pseudo maximum likelihood estimation of structural gravity models. International Economics 161, 188-198.

Polenske, Karen R., 1963. An empirical test of interregional input-output models: estimation of 1963 Japanese production. American Economic Review 60(2), Papers and Proceedings of the Eighty-second Annual Meeting of the American Economic Association, 76-82.

Saia, Alessandro, 2017. Choosing the open sea: The cost to the UK of staying out of the euro. Journal of International Economics 108, 82-98.

Santos Silva, João, and Silvana Tenreyro. 2006. The log of gravity. The Review of Economics and statistics 88(4), 641-658.

Sellner, Richard, 2019. Non-discriminatory trade policies in panel structural gravity models: Evidence from Monte Carlo simulations. Review of International Economics 27(3), 854-887.

Smith, Tony E., 1987. Poisson gravity models of spatial flows. Journal of Regional Science 27(3), 315-340.

Van der Veer, Koen, 2015. The private credit insurance effect on trade. Journal of Risk and Insurance 82(3), 601-624.

Weidner, Martin, and Thomas Zylkin, 2021. Bias and consistency in three-way gravity models. Journal of International Economics 132, 103513.

Zhang, Yahua, and Anming Zhang, 2016. Determinants of air passenger flows in China and gravity model: deregulation, LCCs, and high-speed rail. Journal of Transport Economics and Policy 50(3), 287-303.