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Exchange rate regimes

The foreign exchange (FX) rate regime of a country determines how it manages its currency wrt foreign currencies. It can be

- *floating*: fluctuation based on market forces,
- *pegged*: limited flexibility when compared with a basket of currencies or a single currency,
- *fixed*: direct convertibility to another currency.

**Problem:** The *de facto* and *de jure* FX regime in operation in a country often differ.

⇒ Interest in methods for data-driven classification of FX regimes (see e.g., Reinhart and Rogoff, 2004; Levy-Yeyati and Sturzenegger, 2005; Klein and Shambaugh, 2008).
Exchange rate regimes

State of the art: No widely accepted solution. Various classification schemes for de facto FX regimes yielding differing results.

Our approach: Use parametric regression model for exchange rates (also called Frankel-Wei regression) and develop inferential framework.

Contribution: Unified inferential framework for estimating exchange rate regressions and assessing structural changes in them over time.
The Chinese exchange rate regime

China gave up on a fixed exchange rate to the US dollar (USD) on 2005-07-21. The People’s Bank of China announced that the Chinese yuan (CNY) would no longer be pegged to the USD but to a basket of currencies with greater flexibility.

This generated a lot of interest, both in the media and the scientific literature. Initially, little support could be found for these announcements (Frankel and Wei 2007).

Shah, Zeileis, Patnaik (2005) investigate the Chinese de facto FX regime based on exchange rate regression using structural change methods.
Exchange rate regression

The popular workhorse for de facto FX regime classification is a linear regression model suggested by Haldane and Hall (1991) and Frankel and Wei (1994). It is based on log-returns of cross-currency exchange rates (with respect to some floating reference currency).

For China:

\[ \text{CNY}_i = \beta_1 + \beta_2 \cdot \text{USD}_i + \beta_3 \cdot \text{JPY}_i + \beta_4 \cdot \text{EUR}_i + \beta_5 \cdot \text{GBP}_i + u_i, \]

- \( u_i \) is the error term with variance \( \sigma^2 \),
- ISO 4217 abbreviations denote currency returns computed from prices in CHF.
Exchange rate regression

Ordinary least squares (OLS) estimation based on data up to 2005-10-31 \( (n = 68) \) shows that a plain USD peg is still in operation.

\[
\text{CNY}_i = 0.005 + 0.9997 \text{USD}_i + 0.005 \text{JPY}_i \\
- 0.014 \text{EUR}_i - 0.008 \text{GBP}_i + \hat{u}_i
\]

Only the USD coefficient is significantly different from 0 (but not from 1).

The error standard deviation is tiny with \( \hat{\sigma} = 0.028 \) leading to \( R^2 = 0.998 \).
Questions:

1. Is this model for the period 2005-07-26 to 2005-10-31 stable or is there evidence that China kept changing its FX regime after 2005-07-26? (testing)

2. Depending on the answer to the first question:

- Does the CNY stay pegged to the USD in the future (starting form November 2005? (monitoring)
- When and how did the Chinese FX regime change? (dating)
Regime stability

**In practice:** Regressions on various subsets are often used to answer these questions by tracking the evolution of the FX regime in operation, e.g.,

- regressions on splitted samples,
- rolling regressions,
- Kalman filtering.

**More formally:** Structural change techniques can be adapted to the FX regression to estimate and test the stability of FX regimes ⇒ formal inferential framework for assessing if/when/how the coefficients in the FX regression change.
Regime stability

**Problem:** Unlike many other linear regression models, the stability of the error variance (fluctuation band) is of interest as well.

**Solution:** Employ an (approximately) normal regression estimated by ML where the variance is a full model parameter.
Regime stability

The FX regression is essentially a standard linear regression model

\[ y_i = x_i^\top \beta + u_i \]

with coefficients \( \beta \) and error variance \( \sigma^2 \).

The corresponding estimating functions for the parameters are

\[ \psi_\beta(y, x, \beta) = (y - x^\top \beta) x, \]
\[ \psi_{\sigma^2}(y, x, \beta, \sigma^2) = (y - x^\top \beta)^2 - \sigma^2. \]

To test the stability of the parameters \( \beta \) and \( \sigma^2 \), it can be assessed whether the empirical estimating functions \( \hat{\psi}_i \) differ systematically from their zero mean.
Testing

To capture systematic deviations the empirical fluctuation process of scaled cumulative sums of empirical estimating functions is computed:

\[ efp(t) = \hat{B}^{-1/2} n^{-1/2} \sum_{i=1}^{[nt]} \hat{\psi}_i \quad (0 \leq t \leq 1). \]

- theoretical limiting process is the Brownian bridge (FCLT),
- choose boundaries which are crossed by the limiting process (or some functional of it) only with a known probability \( \alpha \).
- if the empirical fluctuation process crosses the theoretical boundaries the fluctuation is improbably large \( \Rightarrow \) reject the null hypothesis.
Testing
Testing

This corresponds to using a double maximum statistic

$$\max_{j=1,...,k} \max_{i=1,...,n} |efp_j(i/n)|$$

which is 1.078 here ($p = 0.73$).

Alternatives:

- Andrews’ supLM test,
- Score-based MOSUM test,
- Nyblom-Hansen test (with Cramér-von Mises functional),

which also fall into this framework (Zeileis, 2005). The latter two are more suitable for capturing multiple changes.
Monitoring

The same ideas can be used to test whether incoming observations $i > n$ conform with an established model.

**Basic assumption:** The model parameters are stable in the history period $i = 1, \ldots, n$.

The same empirical fluctuation process $efp(t)$ is updated in the monitoring period and suitable boundaries can again be derived (Zeileis, 2005).
Monitoring

This signals a clear increase in the error variance which is picked up by the monitoring procedure on 2006-03-27.

The regression coefficients did not change significantly, signalling that a USD peg is still in operation.

Using data from the extended period up to 2007-11-29, we fit a segmented model to determine where and how the model parameters changed.
Bai and Perron (2003) describe a strategy for estimating the breakpoints in a linear regression based on the residual sum of squares (RSS).

For the additive objective function RSS, a dynamic programming algorithm that evaluates all potential $m$-partitions (i.e., with $m$ breakpoints) is available. It is an application of Bellman’s principle of optimality.

**Problem:** Dating based on the RSS does not exploit changes in the error variance (only regression coefficients).
For the FX regression, we employ the same dynamic programming algorithm based on a different additive objective function: the (negative) log-likelihood from a normal model \( \Rightarrow \) changes in the variance are also captured.

For a fixed given number of breaks \( m \), the optimal breaks (wrt log-likelihood) can be found. To determine the number of breaks, standard techniques for model selection can be applied here, e.g., information criteria or sequential tests.

Sometimes, these do not work well out of the box, but should be handled with care and enhanced by other techniques.
Dating

![Graph showing the relationship between the number of breakpoints and negative log-likelihood. The graph has two lines: one labeled LWZ and the other labeled neg. Log-Lik. The LWZ line increases with the number of breakpoints, while the neg. Log-Lik. line decreases.](image-url)
The estimated breakpoint (maximizing the segmented likelihood) is 2006-03-14.

The corresponding parameter estimates are

<table>
<thead>
<tr>
<th>start/end</th>
<th>$\beta_0$</th>
<th>$\beta_{USD}$</th>
<th>$\beta_{JPY}$</th>
<th>$\beta_{EUR}$</th>
<th>$\beta_{GBP}$</th>
<th>$\sigma$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005-07-26</td>
<td>-0.005</td>
<td>0.999</td>
<td>0.005</td>
<td>-0.015</td>
<td>0.007</td>
<td>0.028</td>
<td>0.998</td>
</tr>
<tr>
<td>2006-03-14</td>
<td>(0.002)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.017)</td>
<td>(0.008)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2006-03-15</td>
<td>-0.019</td>
<td>0.979</td>
<td>-0.014</td>
<td>0.006</td>
<td>0.004</td>
<td>0.092</td>
<td>0.962</td>
</tr>
<tr>
<td>2007-11-29</td>
<td>(0.004)</td>
<td>(0.013)</td>
<td>(0.011)</td>
<td>(0.029)</td>
<td>(0.016)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

and correspond to a

- very tight USD peg,
- slightly relaxed USD peg (with some more appreciation).
India also has an expanding economy with a currency receiving increased interest over the last years. We track the evolution of the INR FX regime since trading in the INR began.

Using weekly returns from 1993-04-09 through to 2007-11-30 (yielding \( n = 765 \) observations), we fit a single FX regression using the same basket as above.

As we would expect multiple changes, we assess its stability with the Nyblom-Hansen test, leading to a test statistic of 3.19 (\( p < 0.005 \)). Alternatively, a MOSUM test could be used. The double maximum test has less power, resulting in a test statistic of 1.732 (\( p = 0.029 \)).
Application: Indian FX regimes

Time
JPY
(Variance)
USD
GBP
(Intercept)
DUR
(Intercept)
Application: Indian FX regimes

![Graph showing the relationship between the number of breakpoints and LWZ negative Log-Likelihood.](image-url)
## Application: Indian FX regimes

Dating finds the following FX regimes:

<table>
<thead>
<tr>
<th>start/end</th>
<th>$\beta_0$</th>
<th>$\beta_{USD}$</th>
<th>$\beta_{JPY}$</th>
<th>$\beta_{DUR}$</th>
<th>$\beta_{GBP}$</th>
<th>$\sigma$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1993-04-09</td>
<td>-0.006</td>
<td>0.972</td>
<td>0.023</td>
<td>0.011</td>
<td>0.020</td>
<td>0.157</td>
<td>0.989</td>
</tr>
<tr>
<td>1995-03-03</td>
<td>(0.017)</td>
<td>(0.018)</td>
<td>(0.014)</td>
<td>(0.032)</td>
<td>(0.024)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1995-03-10</td>
<td>0.161</td>
<td>0.943</td>
<td>0.067</td>
<td>-0.026</td>
<td>0.042</td>
<td>0.924</td>
<td>0.729</td>
</tr>
<tr>
<td>1998-08-21</td>
<td>(0.071)</td>
<td>(0.074)</td>
<td>(0.048)</td>
<td>(0.155)</td>
<td>(0.080)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1998-08-28</td>
<td>0.019</td>
<td>0.993</td>
<td>0.010</td>
<td>0.098</td>
<td>-0.003</td>
<td>0.275</td>
<td>0.969</td>
</tr>
<tr>
<td>2004-03-19</td>
<td>(0.016)</td>
<td>(0.016)</td>
<td>(0.010)</td>
<td>(0.034)</td>
<td>(0.021)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2004-03-26</td>
<td>-0.047</td>
<td>0.739</td>
<td>0.117</td>
<td>0.463</td>
<td>0.135</td>
<td>0.579</td>
<td>0.792</td>
</tr>
<tr>
<td>2007-11-30</td>
<td>(0.043)</td>
<td>(0.045)</td>
<td>(0.043)</td>
<td>(0.118)</td>
<td>(0.059)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. tight USD peg,
2. flexible USD peg,
3. tight USD peg,
4. flexible USD peg with weight on JPY, EUR, GBP.
Software

All methods are implemented in the R system for statistical computing and graphics

http://www.R-project.org/

in the contributed packages **strucchange** and **fxregime**, available from CRAN. **fxregime** is under development at R-Forge.

http://CRAN.R-project.org/
http://R-Forge.R-project.org/
Summary

- Analysis of de facto exchange rate regimes is embedded in a formal inferential framework.
- Both, the coefficients (currency weights) and the error variance (fluctuation band) can be assessed by adopting an approximately normal model.
- Empirical estimating equations can be assessed in historical samples (testing) or online in incoming data (monitoring).
- Based on the corresponding likelihood the model can be optimally segmented (given the number of breakpoints).
- Determining the number of breakpoints is sometimes not straightforward.
- Testing and monitoring are computationally cheap, dating is more costly.
References


